MATH 112
Final Exam
Winter 2017

Name ______________________________

Student ID #______________________ Section ______________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	16	
2	18	
3	12	
4	12	
5	16	
6	12	
7	14	
Total	100	

- Your exam should consist of this cover sheet, followed by 7 problems on 7 pages. Check that you have a complete exam.
- You are allowed to use a TI30-XIIS calculator, a ruler, and one sheet of hand-written notes. All other sources are forbidden.
- Do not use scratch paper. If you need more room, use the back of the page and indicate to the grader you have done so.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- Unless otherwise indicated, you must use the methods of this course and show all of your work. Clearly label lines and points that you are using and show all calculations. The correct answer with little or no supporting work may result in no credit. If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Unless otherwise indicated, you may round your **final answer** to two digits after the decimal.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

GOOD LUCK!

1. (16 points) Compute the indicated derivative. DO NOT SIMPLIFY.

   (a) $h(r,t) = te^{3r} - \dfrac{1}{r^4} + \dfrac{\ln(t)}{8}$

   $h_r(r,t) =$

   $h_t(r,t) =$

   (b) $f(x,y) = (x^2y + xy^2)^6$

   $f_x(x,y) =$

   (c) $z = y^5 e^{(x^2+4y)}$

   $\dfrac{\partial z}{\partial y} =$

2. (18 points) You manufacture a product using two liquid materials: Mat 1 and Mat 2.

   Using x gallons of Mat 1 and y gallons of Mat 2, you produce

   $$P(x, y) = 6x^2 + 5.1y^2 - 0.2x^3 - 0.04y^3 \text{ bottles of your product.}$$

   (a) How many bottles do you produce if you use 10 gallons of Mat 1 and 50 gallons of Mat 2?

   ANSWER: ____________________bottles

   (b) The production function $P(x, y)$ has two critical points, the origin $(0, 0)$ and one other. The critical point with positive values of x and y yields the maximum production.

   How much of each material is required to maximize production?

   ANSWER: ____________________gallons of Mat 1

   ____________________gallons of Mat 2

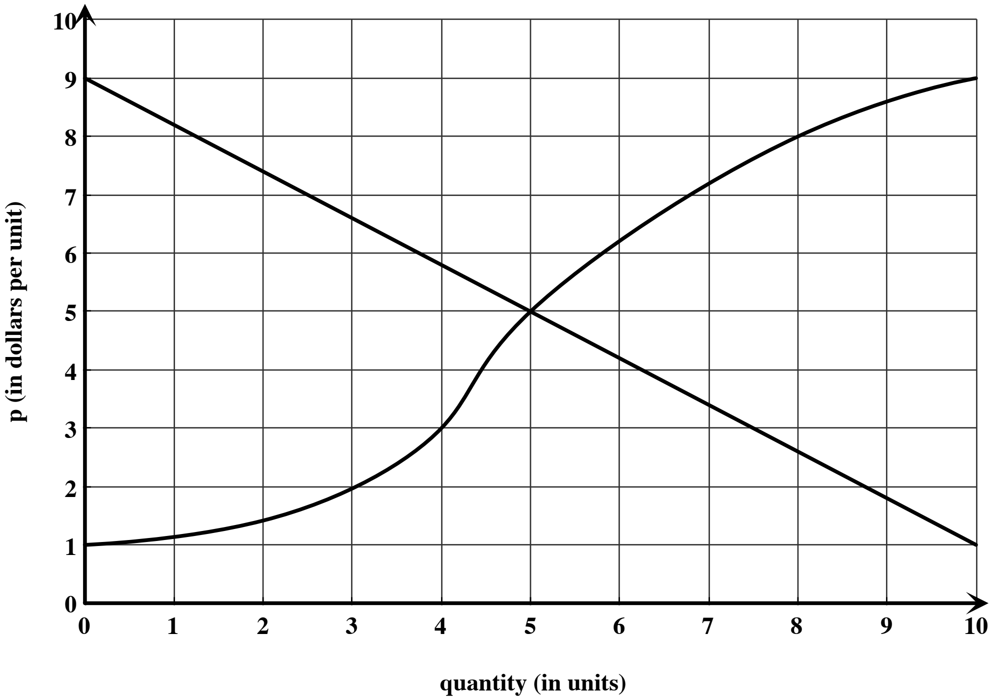
   (c) Use a partial derivative to estimate the **change in production** if you use 25 gallons of Mat 1 and the amount of Mat 2 used increases from 70 to 71 gallons.

   ANSWER: ____________________bottles

   (d) If you use 10 gallons of Mat 1 and want to produce 400 bottles of your product, how many gallons of Mat 2 do you need? (The amount must be greater than 0 gallons.)

   ANSWER: ____________________gallons of Mat 2

3. (12 points) The graph below shows supply and demand for a certain product.



(a) On the graph above, label the demand curve, the supply curve, and the equilibrium point.

(b) Use the graph to compute consumer surplus and producer surplus at equilibrium. Show all your work.

ANSWER: consumer surplus = $____________________

ANSWER: producer surplus = $____________________

4. (12 points) You sell Things. The formulas below are for marginal revenue and marginal cost, where q is measured in Things and MR and MC are in dollars per Thing.

$$MR(q) = 540 \text{ and } MC(q) = 27\sqrt{q+16}.$$

Fixed Costs are \$2500.

Compute the maximum possible profit.

ANSWER: ____________________dollars

5. (16 points) Water is flowing into and out of two vats. After t minutes, the amount of water in Vat A is

$$A(t) = 200t + 3364 \text{ gallons.}$$

The amount of water in Vat B after t minutes is $B(t)$ gallons. You do not know the formula for $B(t)$ but you know that

$$B'(t) = -100t^2 + 1440t + 816.$$

(a) When is the **water level** in Vat B at a maximum?

ANSWER: $t =$ ____________________ minutes

(b) When is the water level in Vat B **rising most rapidly**?

ANSWER: $t =$ ____________________ minutes

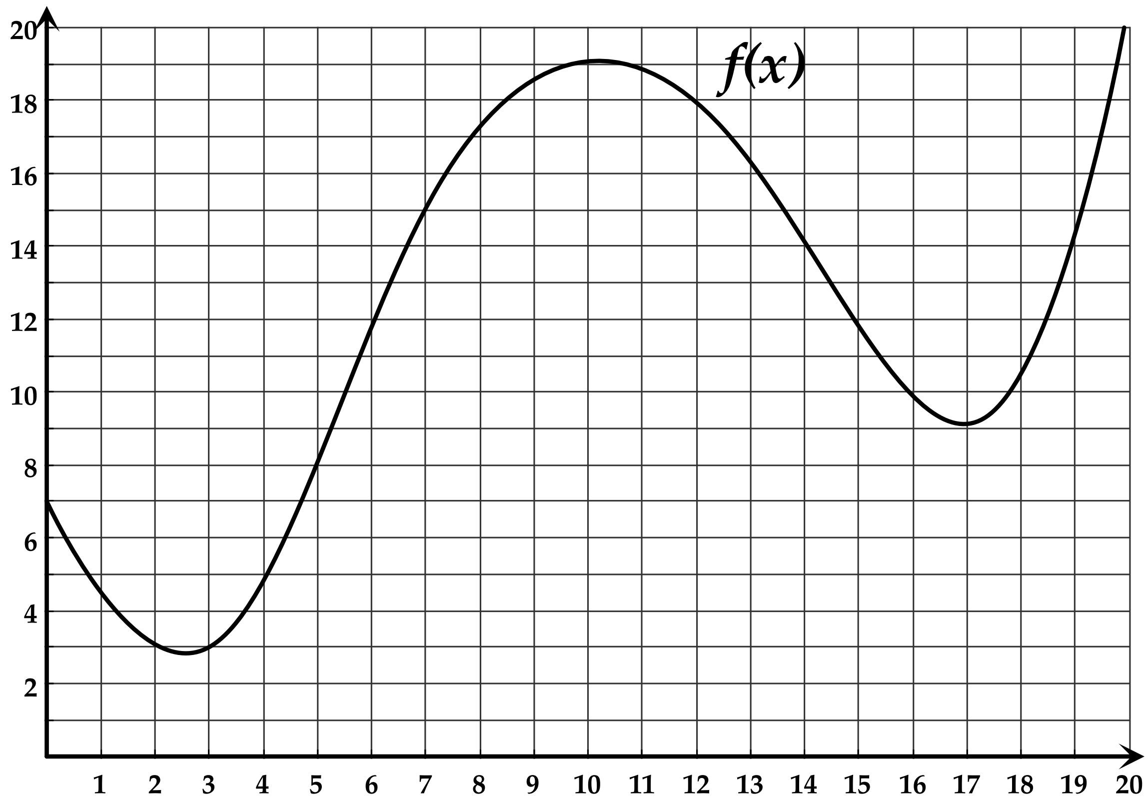
(c) At $t = 6$, Vat B contains 20,000 gallons more than Vat A. How much water does Vat B contain at $t = 0$?

ANSWER: ____________________ gallons

(d) Let $D(t) = B(t) - A(t)$. Give the longest interval on which $D(t)$ is increasing.

ANSWER: from $t =$ ____________________ to $t =$ ____________________ minutes

6. (12 points) The graph below shows a function $y = f(x)$.



(a) Approximate the value of $f'(3)$.

ANSWER: $f'(3) =$ ____________________

(b) Approximate the value of $\int_5^6 f(x)\,dx$.

ANSWER: $\int_5^6 f(x)\,dx =$ ____________________

(c) Over which of the following intervals does $f'(x)$ change from positive to zero to negative? Select all that apply.

_______ From $x = 1$ to $x = 4$.
_______ From $x = 7$ to $x = 13$.
_______ From $x = 11$ to $x = 17$.
_______ From $x = 16$ to $x = 18$.

(d) Over which of the following intervals is $f'(x)$ increasing? Select all that apply.

_______ From $x = 1$ to $x = 4$.
_______ From $x = 7$ to $x = 13$.
_______ From $x = 11$ to $x = 17$.
_______ From $x = 16$ to $x = 18$.

7. (14 points) There is a function $f(x)$ whose formula you do not know. You know that

$$f(x+h) - f(x) = \frac{2h}{(2x+2h+3)(2x+3)}.$$

(a) Compute $f(1.05) - f(1)$.
(Give at least three digits after the decimal in your final answer.)

ANSWER: ____________________

(b) Find the average rate of change of $f(x)$ from $x = 1$ to $x = 1.05$.
(Give at least three digits after the decimal in your final answer.)

ANSWER: ____________________

(c) Compute $f'(0.25)$.
(Give at least three digits after the decimal in your final answer.)

ANSWER: $f'(0.25) =$____________________