1. (12 pts) Use derivatives and anti-derivatives to compute the following:

(a) Let $f(x) = \dfrac{\ln(12x+3)}{4x+2}$. Find the slope of the tangent line to $f(x)$ at $x = 0$. (Give your answer accurate to 3 digits after the decimal).

$$f'(x) = \frac{(4x+2)\frac{12}{12x+3} - \ln(12x+3)\cdot 4}{(4x+2)^2}$$

$$f'(0) = \frac{2\cdot\frac{12}{3} - \ln(3)\cdot 4}{(2)^2} = \frac{8-4\ln(3)}{4} = 2-\ln(3) \approx 0.901$$

$$\boxed{f'(0) = 2-\ln(3) \approx 0.901}$$

(b) Let $TC(x) = 50 + 12x^2e^{x/2}$ dollars where x is in items. Find the marginal cost at $x = 2$ items. (Round your answer to the nearest cent).

$$TC'(x) = 12x^2\cdot\frac{1}{2}e^{x/2} + 24x\,e^{x/2}$$

$$TC'(2) = 48\cdot\frac{1}{2}e^1 + 48e^1$$

$$= 24e + 48e = 72e \approx 195.72$$

$$\boxed{MC(2) = 72e \approx 195.72}$$

(c) Find the general anti-derivative: $\int \frac{2}{\sqrt{x^3}} + \frac{3}{5x}\,dx$. Put a box around your final answer.

$$= \int 2x^{-3/2} + \frac{3}{5}\frac{1}{x}\,dx$$

$$= 2\frac{1}{(-\frac{1}{2})}x^{-1/2} + \frac{3}{5}\ln(x) + C = \boxed{-4x^{-1/2} + \frac{3}{5}\ln(x) + C}$$

(d) Evaluate $\int_0^1 x^2(8x-3) + 4e^{2x}\,dx$. Put a box around your final answer.

$$\int_0^1 8x^3 - 3x^2 + 4e^{2x}\,dx = 2x^4 - x^3 + 2e^{2x}\Big|_0^1$$

$$= (2-1+2e^2) - (0-0+2)$$

$$= 1+2e^2-2$$

$$= \boxed{2e^2 - 1 \approx 13.778}$$

2. (13 pts) You sell Things. The functions for marginal revenue and average cost (both in dollars/item) are given by

$$MR(q) = 50 - 2q \text{ and } AC(q) = \frac{30}{q} + 2 + q, = 30q^{-1} + 2 + q$$

where q is in **thousands** of items.
Keep enough digits to be accurate to the nearest Thing and nearest dollar.

(a) Is Total Revenue concave up, concave down, or neither at $q = 4$ items?
(Show some work/calculations to justify your answer)

$TR'(q) = 50 - 2q$

$TR''(q) = -2 \leftarrow$ NEGATIVE (EVERYWHERE, INCLUDING 4)

Circle One: CONCAVE UP or $\boxed{\text{CONCAVE DOWN}}$ or NEITHER

(b) Find the one positive critical value for Average Cost and use either the 1st derivative number line or the second derivative test to determine if it gives a local maximum, local minimum, or neither (clearly show your reasoning).

$AC'(q) = -30q^{-2} + 1 \overset{?}{=} 0 \Rightarrow 1 = \frac{30}{q^2} \Rightarrow q^2 = 30$

$q = \sqrt{30} \approx 5.477$

1st derivative number line

AC $\searrow$ | $\nearrow$

$AC' < 0$ $\sqrt{30}$ $AC' > 0$

or

$AC''(q) = 60q^{-3}$ WHICH IS POSITIVE (CONCAVE UP)

The critical point $q =$ 5.477 thousand Things gives a

(CIRLCE ONE): $\boxed{\text{LOCAL MIN}}$ or LOCAL MAX or NEITHER

(c) Find the maximum profit.

$\boxed{TC(q) = 30 + 2q + q^2 \Rightarrow mc(q) = 2 + 2q}$ & $\boxed{TR(q) = 50q - q^2}$

$mr = mc \Rightarrow 50 - 2q = 2 + 2q \Rightarrow 48 = 4q \Rightarrow q = 12$

$P(12) = TR(12) - TC(12)$

$= (50(12) - (12)^2) - (30 + 2(12) + (12)^2)$

$= 456 - 198 = 258$

$\boxed{258 \text{ thousand dollars}}$

3. (12 pts) The amount of water in two vats is changing. The amount of water (in gallons) in Vat A and in Vat B are given by $A(t)$ and $B(t)$ respectively, where t is in hours. You are told that the vats start with the same amount of water and that

Vat A <u>RATE</u> of change:	$A'(t) = -3t^2 + 18t - 15$	gallons/hour
Vat B <u>AMOUNT</u>:	$B(t) = -t^2 + 8t + 30$	gallons

(a) Find the formula for $A(t)$ without any undetermined constants.
(Hint: the problem told you $A(0) = B(0)$).

$$A(t) = -t^3 + 9t^2 - 15t + C$$

$$A(0) = B(0) = 30 \Rightarrow C = 30$$

$\boxed{A(t) = -t^3 + 9t^2 - 15t + 30}$

(b) Find all times at which $A(t)$ has a point of inflection.

$$A'(t) = -3t^2 + 18t - 15$$

$$A''(t) = -6t + 18 \overset{?}{=} 0$$

$$t = 3$$

$\cup$ $\cap$

$A'' > 0$ 3 $A'' < 0$

$\boxed{t = 3}$ hours

(c) What is the highest *amount* in Vat A during the interval from $t = 0$ to $t = 7$ hours?

$$A'(t) = -3t^2 + 18t - 15 \overset{?}{=} 0 \Rightarrow -3(t^2 - 6t + 5) = 0$$

$$-3(t-1)(t-5) = 0$$

$$t = 1,\ t = 5$$

$$A(0) = 30$$

$$A(1) = -1 + 9 - 15 + 30 = 23$$

$$A(5) = -125 + 9(25) - 15(5) + 30 = 55$$

$$A(7) = -7^3 + 9(49) - 15(7) + 30 = 23$$

$\boxed{55}$ gallons

(d) What is the highest *rate* of change in Vat B on the interval $t = 0$ to $t = 7$? (*i.e.* level is rising most rapidly)

$$B'(t) = \underbrace{-2t + 8}_{} \leftarrow$$ WANT MAX OF THIS ON $t=0$ TO $t=7$

$$-2 \overset{?}{=} 0$$ NEVER (NO CRITICAL PTS)

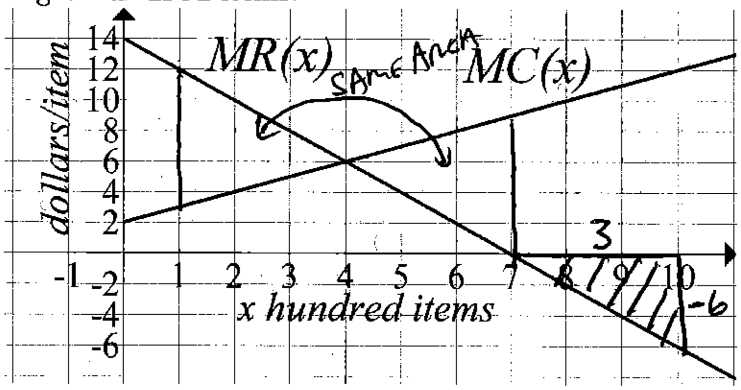
MUST BE AT ENDPOINTS

$$B'(0) = 8$$

$$B'(7) = -14 + 8 = -6$$

$\boxed{8}$ gallons/hour

4. (13 pts) The graph below shows marginal revenue and marginal cost (in dollars per item) for producing and selling x **hundred** items.



You are also told that **Fixed Costs** are $FC = \$1050$ (10.5 hundred dollars). Use the picture to estimate the answers to the questions below *as accurately as possible.*

(a) For the 3 quick questions below, fill in the blanks:

i. Total Revenue is maximized at $x =$ 7 hundred items

ii. Profit is maximized at $x =$ 4 hundred items

iii. Marginal Revenue is maximized at $x =$ 0 hundred items

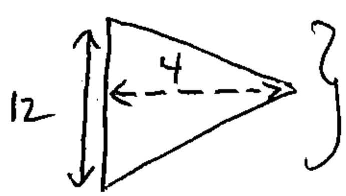
(b) Estimate the following from the graph:

i. $\int_7^{10} MR(x)\,dx = \frac{1}{2}(3)(-6) = \boxed{-9}$ hundred dollars

ii. $TR''(3) = MR'(3) =$ "$\underbrace{\text{slope of MR at 3}}$" $= \frac{14-0}{0-7} = \boxed{-2}$

A LINE! (0,14), (7,0)
USE 2 PTS & GET SLOPE

(c) Estimate the maximum profit.



AREA $= \frac{1}{2}(12)(4) = 24$

$P(4) = \underbrace{P(0)}_{-10.5} + 24$
$= -10.5 + 24 = 13.5$

Max Profit = 13.5 hundred dollars

(d) There are two quantities when profit is zero. Find them both. (Hint: Think very carefully, take your time, and remember that profit starts at -10.5 hundred dollars)

AREA FROM 0 TO 1 ≈ "5 + 1/4 BOXES" ≈ 10.5 hundred dollars
So $P(1) = -10.5 + 10.5 = 0$ THEN AGAIN AT $x = 7$

$x \approx$ 1 and $x \approx$ 7 hundred items