Math 112 - Winter 2019
Exam 2
March 5, 2019

Name: _____

Section: _____

Student ID Number: _____

1	12	
2	13	
3	12	
4	13	
Total	50	

- After this cover page, there are 4 problems spanning 4 pages. Please make sure your exam contains all of this material.

- You are allowed to use a Ti-30x IIS Calculator model ONLY (**no other calculators allowed**). And you are allowed one **hand-written** 8.5 by 11 inch page of notes (front and back).

- You must show your work on all problems. The correct answer with no supporting work may result in no credit.

- If you use a guess-and-check, or calculator, method when an algebraic method is available, you may not receive full credit.

- If you need more room, use the backs of the pages and indicate to the grader that you have done so.

- Raise your hand if you have a question.

- There are **multiple versions** of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the academic misconduct board. Sit far away from your study partners and keep your eyes down, don't risk a zero on this exam!

- You have 50 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

GOOD LUCK!

1. (12 pts) Use derivatives and anti-derivatives to compute the following:

(a) Let $f(x) = \dfrac{\ln(12x + 3)}{4x + 2}$. Find the slope of the tangent line to $f(x)$ at $x = 0$.
   (Give your answer accurate to 3 digits after the decimal).

$$f'(0) = \underline{\hspace{3cm}}$$

(b) Let $TC(x) = 50 + 12x^2 e^{x/2}$ dollars where x is in items. Find the marginal cost at $x = 2$ items. (Round your answer to the nearest cent).

$$MC(2) = \underline{\hspace{3cm}}$$

(c) Find the general anti-derivative: $\displaystyle\int \frac{2}{\sqrt{x^3}} + \frac{3}{5x} \, dx$. Put a box around your final answer.

(d) Evaluate $\displaystyle\int_0^1 x^2(8x - 3) + 4e^{2x} \, dx$. Put a box around your final answer.

2. (13 pts) You sell Things. The functions for marginal revenue and average cost (both in dollars/item) are given by

$$MR(q) = 50 - 2q \ \text{ and } \ AC(q) = \frac{30}{q} + 2 + q,$$

where q is in **thousands** of items.
Keep enough digits to be accurate to the nearest Thing and nearest dollar.

(a) Is **Total Revenue** concave up, concave down, or neither at $q = 4$ items?
(Show some work/calculations to justify your answer)

Circle One: CONCAVE UP or CONCAVE DOWN or NEITHER

(b) Find the one positive critical value for Average Cost and use either the 1st derivative number line or the second derivative test to determine if it gives a local maximum, local minimum, or neither (clearly show your reasoning).

The critical point $q = $ _____ thousand Things gives a

(CIRLCE ONE): LOCAL MIN or LOCAL MAX or NEITHER

(c) Find the maximum profit.

_____ thousand dollars

3. (12 pts) The amount of water in two vats is changing. The amount of water (in gallons) in Vat A and in Vat B are given by $A(t)$ and $B(t)$ respectively, where t is in hours. You are told that the vats start with the same amount of water and that

$$\text{Vat A } \underline{\text{RATE}} \text{ of change:} \quad A'(t) = -3t^2 + 18t - 15 \qquad \text{gallons/hour}$$
$$\text{Vat B } \underline{\text{AMOUNT}}: \quad B(t) = -t^2 + 8t + 30 \qquad \text{gallons}$$

(a) Find the formula for $A(t)$ without any undetermined constants.
   (Hint: the problem told you $A(0) = B(0)$).

$$A(t) = \underline{\hspace{4cm}}$$

(b) Find all times at which $A(t)$ has a point of inflection. (Justify your answer by drawing the 2nd deriv. number line, indicating concavity, as we have done in class).
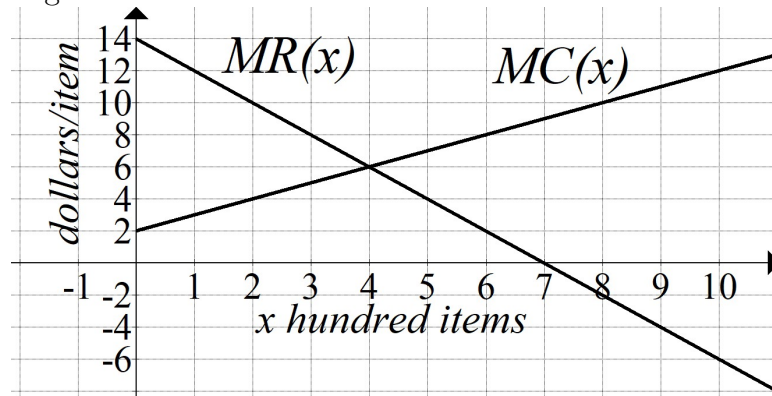
$$t = \underline{\hspace{3cm}} \text{ hours}$$

(c) What is the highest *amount* in Vat A during the interval from $t = 0$ to $t = 7$ hours?

$$\underline{\hspace{3cm}} \text{ gallons}$$

(d) What is the highest *rate* of change in Vat B on the interval $t = 0$ to $t = 7$? (*i.e.* level is rising most rapidly)

$$\underline{\hspace{3cm}} \text{ gallons/hour}$$

4. (13 pts) The graph below shows marginal revenue and marginal cost (in dollars per item) for producing and selling x **hundred** items.

You are also told that **Fixed Costs** are $FC = \$1050$ (10.5 hundred dollars). Use the picture to estimate the answers to the questions below *as accurately as possible*.

(a) For the 3 quick questions below, fill in the blanks:

   i. Total Revenue is maximized at $x =$ _____ hundred items

   ii. Profit is maximized at $x =$ _____ hundred items

   iii. Marginal Revenue is maximized at $x =$ _____ hundred items

(b) Estimate the following from the graph:

   i. $\displaystyle\int_7^{10} MR(x)\,dx =$

   ii. $TR''(3) =$

(c) Estimate the maximum profit.

Max Profit = _____ hundred dollars

(d) There are two quantities when profit is zero. Find them both. (Hint: Think very carefully, take your time, and remember that profit starts at -10.5 hundred dollars)

$x \approx$ _____ and $x \approx$ _____ hundred items