Math 112 - Winter 2014
Final Exam
March 15, 2014

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

1	10	
2	10	
3	11	
4	10	
5	12	
6	12	
7	12	
8	12	
9	11	
Total	100	

- You are allowed to use a scientific calculator (no graphing calculator and no calculator with calculus abilities) and one hand-written 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check, or calculator, method when an algebraic method is available, you may not receive full credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question. We will only clarify the wording of a question, we cannot and will not comment on your work. So don't raise your hand fishing for answers.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam. Keep your eyes down and on your paper. If we see your eyes wandering we will warn you only once before taking your exam from you.
- You have 3 hours to complete the exam.

GOOD LUCK!

1. (10 pts)

(a) Find $f'(x)$ if $f(x) = \ln(x^3 + 1)e^{5x}$.

(b) Find $g_y(x, y)$ if $g(x, y) = \left(x^2 + \frac{2y^4}{3}\right)^{10} + \ln(x) + 7$.

(c) Find the equation of the tangent line to $h(x) = \frac{\sqrt{x+1}}{2} - \frac{27}{x^2}$ at $x = 3$.

2. (10 pts)

(a) Evaluate $\int \frac{3}{2x} + \frac{5}{3x^2} - \sqrt[3]{x}\,dx$

(b) ~~A new financial company opens up and expects its **monthly** rate of income flow to be given by $r(t) = 3000e^{0.04t}$ dollars per month, where t is months after it opened. Find the total income in the first 3 years of operation.~~

Not Covered in Winter 2025

3. (11 pts) The demand function for a product is $p = 50 - x^2$ and the supply function is $p = 2 + 2x$, where p is the price per unit, in dollars, and x in the number of units.

(a) Find the price and quantity that correspond to market equilibrium.

(b) Find the consumer's surplus.

(c) Find the supplier's surplus.

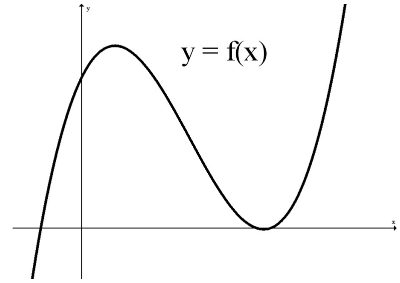
4. (10 pts) Assume the height of a balloon is given by $A(t) = t^2 - 4t + 30$ where t is in minutes and height is in feet.

   (a) Write out and completely simplify the expression $\dfrac{A(t+h) - A(t)}{h}$

   (b) Find the **average** rate of ascent over the interval from $t = 2$ to $t = 5$. (Give units)

   (c) Find the **instantaneous** rate of ascent at $t = 5$. (Give units)

5. (a) (7 pts) Let $f(x) = \frac{4}{3}x^3 - 26x^2 + 88x + 400$.
(A sketch of $f(x)$ is given).



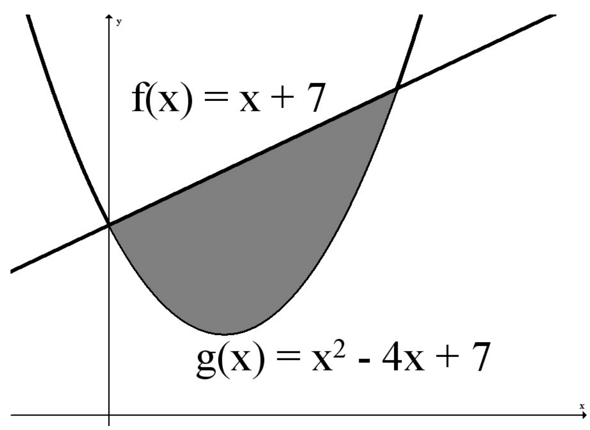
i. Find the longest interval over which $f(x)$ is decreasing.

ANSWER: $x =$ ____________ to $x =$ ____________

ii. Find the x value(s) at which $f(x)$ has a point of inflection.

ANSWER: $x =$ ____________

(b) (5 pts) Find the area of the shaded region between the given curves.
(Hint: First find where the functions intersect.)

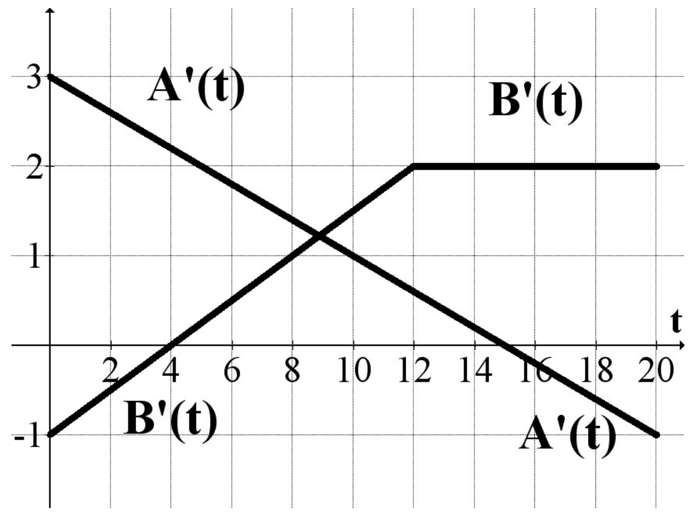


ANSWER: Area = ____________

6. (12 pts)

Let $A(t)$ and $B(t)$ be the functions for the altitude of two Balloons, A and B. The graphs of **RATE OF ASCENT** for each balloon are given, where the t-axis is in minutes and the y-axis is in feet per minute.

You are told that both balloons start at an initial height of 60 ft, so $A(0) = 60$ and $B(0) = 60$. Accurately approximate answers to the following questions from the graph.



(a) (3 pts) Find the altitude of balloon A at time $t = 5$ minutes.

ANSWER: $A(5) =$ ____________ feet

(b) (3 pts) Find the longest interval of time when both the altitude of Balloon A and the altitude of Balloon B are rising (increasing).

ANSWER: $t =$ ________ to $t =$ ________

(c) (3 pts) Let $D(t) = A(t) - B(t)$ be the vertical distance that Balloon A is above B. Find the time and value of $D(t)$ when it is largest.

ANSWER: $t =$ ________ min, LARGEST DIST.= ________ feet

(d) ~~(3 pts) Find the time, t, when balloon B will be 80 feet high.~~
~~(Hint: It might help to first know the height at $t = 12$)~~

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ANSWER: $t =$ ________ min

7. (12 pts) You sell Things. The total cost is given by

$$TC(x) = 3x + \ln(x+1) + 10.$$

The marginal revenue is given by

$$MR(x) = 20 - 2x.$$

In the formulas, x is in thousands of Things, TC is in thousands of dollars, and MR is in dollars per Thing. Remember, as always, that $TR(0) = 0$.

In each problem below, your final answers should have enough digits to be accurate to the nearest Thing, or nearest dollar.

(a) (4 pts) Find the maximum value, in thousands of dollars, of total revenue.

ANSWER: ____________________ thousand dollars

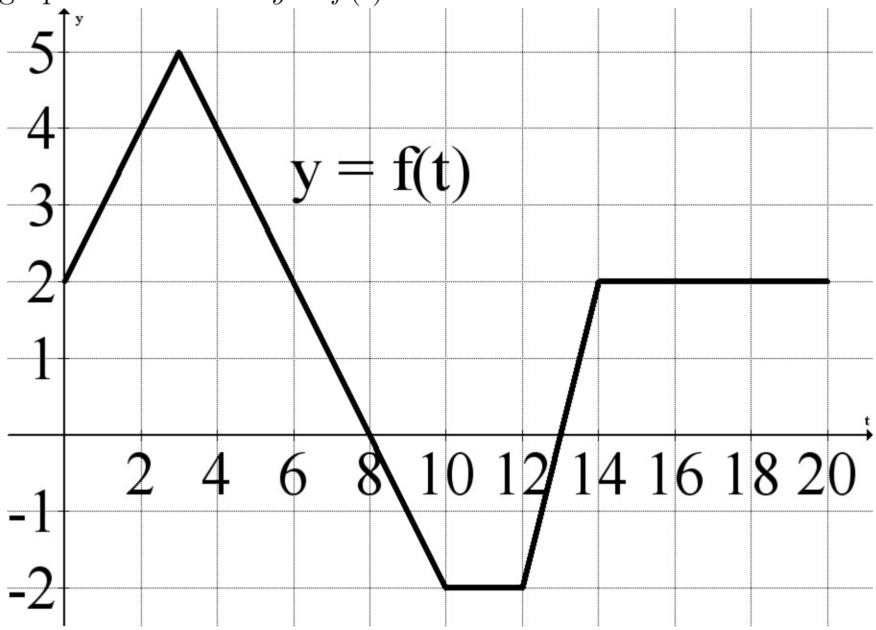
(b) (4 pts) Find the quantity at which profit is maximum.

ANSWER: $x =$ ____________________ thousand Things

(c) (4 pts) Recall that average cost is given by $AC(x) = \frac{TC(x)}{x}$. By making appropriate calculations with $AC(x)$ and its derivatives, determine if $AC(x)$ is increasing, decreasing, or neither at $x = 1$.

ANSWER: (circle one) INCREASING DECREASING NEITHER

8. (12 pts) The graph of a function $y = f(t)$ is below.



Using the graph above, we define a new function

$$A(m) = \int_0^m f(t)\,dt$$

(a) Compute the following:

- $A'(6)$

ANSWER: $A'(6) =$ ____________

- $A''(6)$

ANSWER: $A''(6) =$ ____________

(b) ~~Compute the value of $\int_8^{20} f(t)\,dt$~~

~~ANSWER: $\int_8^{20} f(t)\,dt =$~~ ____________

~~(c) Find all values of m between 0 and 20 at which $A(m)$ has a local minimum.~~

~~ANSWER: $m =$~~ ____________

~~(d) Find the global maximum value of $A(m)$.~~

~~ANSWER: 'Max output from $A(m)$' =~~ ____________

9. (11 pts) Your company produces and sells Seahawks gloves and Mariner hats. In a given month, let x be in hundreds of gloves produced and sold and let y be in hundreds of hats produced and sold. The profit for the month in hundreds of dollars is given by:

$$P(x, y) = 8x + 4xy - 5x^2 - y^2 - 3 \text{ hundred dollars.}$$

(a) Compute the partial derivatives of P.

$P_x(x, y) =$ ________________

$P_y(x, y) =$ ________________

(b) Suppose you currently produce and sell $x = 3$ hundred gloves and $y = 4$ hundred gloves. What would increase your profit more: Selling one more glove or selling one more hat? (Circle one and give a justification that involves partial derivatives)

(c) You are told that the maximum of profit occurs at the critical point. Find the critical point of profit and give the maximum profit value.

Critical point: $(x, y) =$ ________________

'Max output from profit' = ____________ hundred dollars