MATH 112 FINAL EXAM
SPRING 2015

1. (a) HINT: Sketch the graph of $a(t)$, the derivative of the function that gives the level of ammonia at time t. The level is rising when $a(t)$ is positive, which occurs between the two times when $a(t) = 0$.
   ANSWER: from $t = 2$ to $t = 12$ hours
   (b) HINT: Either use the graph of $a(t)$ to notice when $A(t)$ is changing from decreasing to increasing or from increasing to decreasing OR note whether $A''(2) = a'(2)$ and $A''(12) = a'(12)$ are positive or negative and apply the Second Derivative Test.
   ANSWER: A has a local min at $t = 2$ hours and a local max at $t = 12$ hours.
   (c) HINT: Sketch the graph of $b(t) = B'(t)$, a line with positive slope that crosses the t-axis at $t = 10$. When is it at its highest positive value on the interval from 0 to 30?
   ANSWER: $t = 30$ hours
   (d) HINT: Compute $\dfrac{A(9) - A(4)}{9 - 4}$, noting that $A(9) - A(4) = \int_4^9 a(t)\,dt$.
   ANSWER: 68 gallons per hour
   (e) HINT: Set $a(t) = b(t)$ and solve for t. Evaluate either $a(t)$ or $b(t)$ at each of these values of t and see if the rate is positive (which indicates a rising level) or negative (which indicates a falling level).
   ANSWER: At $t = 1.46$ hours, the levels are both falling. At $t = 11.87$ hours, the levels are both rising.

2. (a) HINT: Read the graph of MC to find $MC(18)$.
   ANSWER: 5 dollars
   (b) HINT: TR is maximized at the quantity where MR changes from positive to negative. Maximum TR is the area under the MR graph from $q = 0$ to $q = 18$.
   ANSWER: 108 thousand dollars
   (c) HINT: Compute the area *between* MR and MC from 0 to 3. This gives $TR(3) - VC(3) = TR(3) - (TC(3) - FC) = P(3) + FC$. Solve for FC.
   ANSWER: 26.25 thousand dollars
   (d) HINT: Profit is maximized at $q = 12$ since $MR(12) = MC(12)$. The area *between* MR and MC from 0 to 12 gives $P(12) + FC$.
   ANSWER: 33.75 thousand dollars

3. (a) HINT: First, determine that equilibrium price is $30e^2$.
   Then $PS = \left(24 \cdot 30e^2\right) - \int_0^{24} 30e^{x/12}\,dx$
   ANSWER: \$3020.06
   (b) HINT: First, determine that equilibrium quantity is $x = 12$.
   Then $CS = \int_0^{12} \dfrac{500}{x+8}\,dx - (12 \cdot 25)$.
   ANSWER: \$158.15

4. (a) i. $\dfrac{P(2, 4.01) - P(2, 4)}{0.01} \approx P_y(2, 4) = 200$
   ii. $\dfrac{P(5, 3) - P(4.99, 3)}{0.01} \approx P_x(5, 3) = 500$

(b) HINT: Compute $P_x(7,8)$ and $P_y(7,8)$ and note which is bigger.
ANSWER: one more Object

5. (a) HINT: $f_x(x,y) = 4y^2 - 100$ and $f_y(x,y) = 8xy + 10y - 30$. Set $f_x = 0$ and $f_y = 0$ and solve the resulting system of equations.
ANSWER: $\left(-\frac{1}{2}, 5\right)$, $(-2, \text{-}5)$

(b) HINT: Create a new function $g(y) = f(1,y) = 9y^2 - 30y + 40$ and sketch its graph, noting its lowest point.
ANSWER: $g\left(\dfrac{5}{3}\right) = 15$

6. (a) HINT: Compute $f''(x)$ and sketch its graph, noting where $f''(x)$ is positive: between the two values of x at which $f''(x) = 0$.
ANSWER: from $x = 1$ to $x = 5$

(b) HINT: First compute $\dfrac{TC(2+h) - TC(2)}{h} = \dfrac{3h+8}{3(1+h)}$ and then note what happens to this expression as you let h get closer and closer to 0.
ANSWER: $MC(2) = \frac{8}{3}$

(c) $R'(w) = 4\left(\dfrac{5}{7w}\right)^3 \cdot \dfrac{5}{7} \cdot \left(-\dfrac{1}{w^2}\right) + w \cdot \dfrac{1}{15w+9} \cdot 15 + \ln(15w+9)$

(d) HINT: $\displaystyle\int_1^5 \frac{t^2+1}{t}\, dt = \int_1^5 t + \frac{1}{t}\, dt$
ANSWER: 13.61