MATH 112
Final Exam
Spring 2015

Name ______________________________

Student ID #______________ Section ______________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	25	
2	15	
3	16	
4	9	
5	15	
6	20	
Total	100	

- Your exam should consist of this cover sheet, followed by 6 problems on 7 pages. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- Unless otherwise indicated, you must use the methods of this course and show all of your work. The correct answer with little or no supporting work may result in no credit. If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Unless otherwise indicated, you may round your **final answer** to two digits after the decimal.
- There are multiple versions of this exam. You've signed an honor statement. Don't cheat.

GOOD LUCK!

1. (25 points) The levels of liquid in two vats are rising and falling. One vat contains ammonia and the **instantaneous rate** (in gallons per hour) at which its level is changing at t hours is given by

$$a(t) = -3t^2 + 42t - 72.$$

The second vat contains bleach and the **amount** of bleach (in gallons) at time t hours is given by

$$B(t) = t^2 - 20t + 105.$$

(a) Give the longest interval during which the level of ammonia is rising.

ANSWER: from $t =$____________ to $t =$____________ hours

(b) The **amount** of ammonia (in gallons) at time t hours is given by a function $A(t)$. Give all times at which the graph of $A(t)$ has a horizontal tangent and determine whether each of those times yields a local maximum or a local minimum of $A(t)$.

ANSWER: A has a local (circle one) max min at $t =$____________ hours

A has a local (circle one) max min at $t =$____________ hours

(c) On the interval from $t = 0$ to $t = 30$ hours, when is the level of bleach rising most rapidly?

ANSWER: $t =$____________ hours

THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.

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Here are those functions again:

$$a(t) = -3t^2 + 42t - 72 \text{ and } B(t) = t^2 - 20t + 105.$$

(d) What is the average rate of flow of ammonia from $t = 4$ to $t = 9$?

ANSWER: ____________________gallons per hour

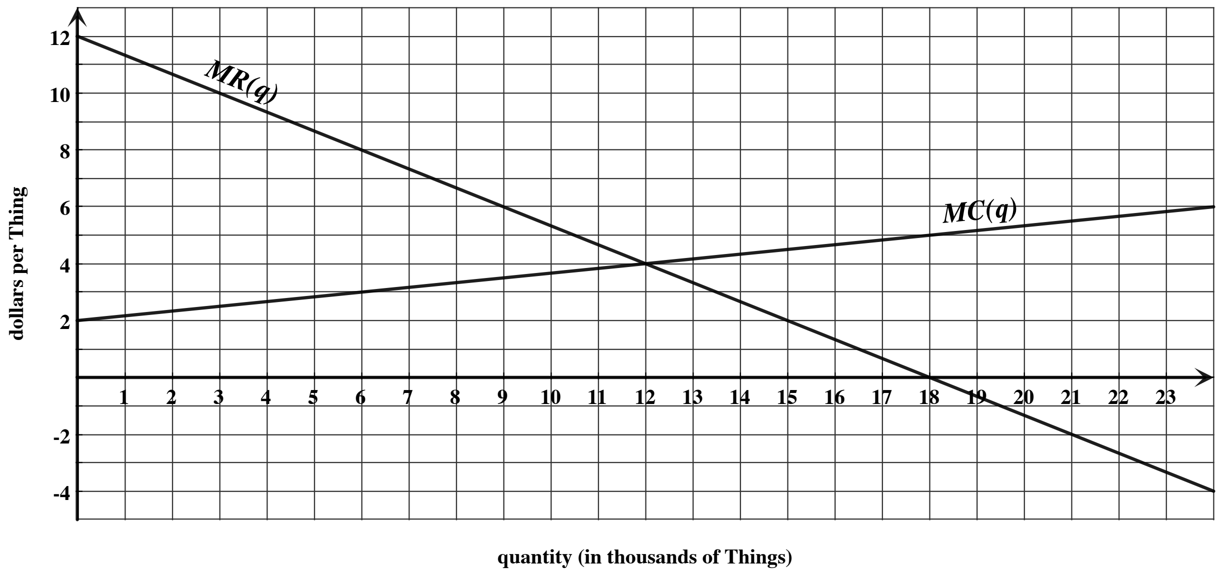
(e) Give all times at which the ammonia and bleach levels have the same instantaneous rate of change. Are the levels rising or falling at those times?

ANSWER:

At $t =$ ______________ hours, the levels are both (circle one) rising falling.

At $t =$ ______________ hours, the levels are both (circle one) rising falling.

2. (15 points) You produce and sell Things. The graphs of marginal revenue and marginal cost are given below. Notice the units on the axes.



(a) What is the cost to produce the $18{,}001^{st}$ Thing? Include units.

ANSWER: ___________________ UNITS: ___________________

(b) What is the maximum possible value of total revenue?

ANSWER: ___________________ thousand dollars

(c) When you produce and sell 3000 Things, profit is 0. Find the value of your fixed cost.

ANSWER: ___________________ thousand dollars

(d) What is the maximum possible value of profit? (Use the fixed cost you found in part (c). If you were unable to get an answer for part (c), use $FC = 10$ to do this part.)

ANSWER: ___________________ thousand dollars

3. (16 points)

(a) The supply function for a product is given by $p = 30e^{x/12}$, where x is the number of units and p is in dollars per unit.

The equilibrium quantity is 24 units. Find the producer's surplus at equilibrium.

(Round your answer to the nearest cent.)

ANSWER: $____________________

(b) The demand function for a different product is given by $p = \dfrac{500}{x+8}$, where x is the number of units and p is in dollars per unit.

The equilibrium price is $25 per unit. Find the consumer's surplus at equilibrium.

(Round your answer to the nearest cent.)

ANSWER: $____________________

4. (9 points) The ABC Corp. makes and sells Items and Objects. If they sell x hundred Items and y hundred Objects, profit (in hundreds of dollars) is given by a function $P(x, y)$.

   We do not have the formula for profit but we have its partial derivatives:

$$P_x(x, y) = 3x^2y + 2xy^3 + 5 \text{ and } P_y(x, y) = x^3 + 3x^2y^2.$$

   (a) Approximate each of the following.

      i. $\dfrac{P(2, 4.01) - P(2, 4)}{0.01}$

      ANSWER: $\dfrac{P(2, 4.01) - P(2, 4)}{0.01} \approx$ ________________

      ii. $\dfrac{P(5, 3) - P(4.99, 3)}{0.01}$

      $\dfrac{P(5, 3) - P(4.99, 3)}{0.01} \approx$ ________________

   (b) If a customer has already ordered 700 Items and 800 Objects and decides to order either one more Item or one more Object, which will lead to a larger increase in profit? Show your work.

      ANSWER: (circle one) one more Item one more Object

5. (15 points) Let $f(x, y) = 4xy^2 + 5y^2 - 100x - 30y + 140$.

   (a) Find all points (x, y) at which f could have a local maximum or local minimum.

   ANSWER: (list all points) ______________________

   (b) If $x = 1$ is fixed, then $g(y) = f(1, y)$ is a function of a single variable, y. What is the smallest value of $g(y)$?

   ANSWER: ______________

6. (20 points)

(a) Find the longest interval on which $f(x) = -x^4 + 12x^3 - 30x^2 - 12x + 15$ is **concave up**.

ANSWER: from $x =$____________ to $x =$____________

(b) If $TC(2+h) - TC(2) = \dfrac{3h^2 + 8h}{3(1+h)}$, what is $MC(2)$?

ANSWER: $MC(2) =$________________

(c) Compute the derivative of $R(w) = \left(\dfrac{5}{7w}\right)^4 + w\ln(15w + 9)$. Do not simplify. Put a box around your answer.

(d) Compute $\displaystyle\int_1^5 \frac{t^2+1}{t}\,dt$. Put a box around your answer.