MATH 112
Final Exam
Spring 2023

Name _____

Student ID #_____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

- Check that your exam contains **6** problems and 1 scratch sheet.
- You are allowed to use a non-graphing scientific calculator, a ruler, and one 8.5 by 11 inch sheet (front and back) of hand-written notes. All other sources are forbidden.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- **You must show your work.** Clearly label lines and points that you are using and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- When rounding is necessary, you may round your final answer to two digits after the decimal.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the last page and **indicate that you have done so**.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.

GOOD LUCK!

1. (16 pts) For each of the following, you do not need to simplify.

(a) Find the derivative of $f(x) = \dfrac{3}{2x^4} - \dfrac{10}{\sqrt{x}} - \dfrac{5x}{3}$.

ANSWER: $f'(x) =$ _____

(b) Find the 1st and 2nd derivatives of $g(x) = \ln(x^3 + 4)$.

ANSWER: $g'(x) =$ _____

ANSWER: $g''(x) =$ _____

(c) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $z = \dfrac{4x}{y} + 5xy^4 - e^{3y} + \ln(x)$.

ANSWERS: $\dfrac{\partial z}{\partial x} =$ _____ $\dfrac{\partial z}{\partial y} =$ _____

2. (17 pts)

(a) Find the equation for the tangent line to $f(x) = 5xe^{3x} + \sqrt{2x+4}$ at $x = 0$.
Reminder: Your answer will be a line, *i.e.* $y = mx + b$. So you need to find the slope and the y-intercept of this line.

ANSWER: $y =$ _____

(b) Find $g'(x)$ if $\dfrac{g(x+h) - g(x)}{h} = \dfrac{6}{x+h} - 4x + x^2 h$

ANSWER: $g'(x) =$ _____

(c) Evaluate the integrals:

i. $\displaystyle\int \left(2e^{-3x} + \frac{x^2 - 5x}{x^2}\right) dx$

ANSWER: _____

ii. $\displaystyle\int_1^3 \left(\frac{18}{x^2} + 2x\right) dx$

ANSWER: _____

3. (18 points) For all parts below, round final answers to two digits after the decimal.

(a) The formulas for two functions are:

$$f(x) = x^2 - 8x + 12 \quad \text{and} \quad g(x) = \frac{4}{3}x^3 - 26x^2 + 88x + 600.$$

i. Find the global maximum and global minimum values of $g(x)$ over the interval $x = 0$ to $x = 8$.

ANSWERS: MIN VALUE = _____ MAX VALUE = _____

ii. Find the longest interval on which $f(x)$ is decreasing and $g(x)$ is decreasing.

ANSWER: $x =$ _____ to $x =$ _____

(b) The function $h(x) = 12\ln(x) - 2x + 6$ has one critical number, find it and identify if whether it corresponds to a local max, a local min, or a horizontal point of inflection.

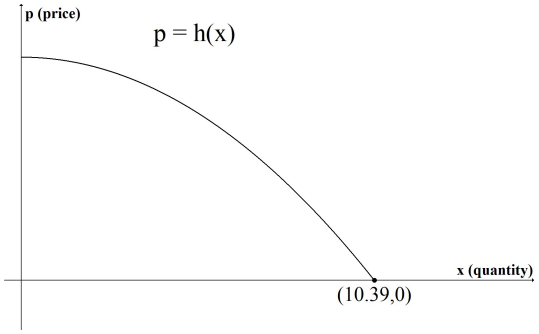
ANSWER: $x =$ _____

Circle one: Local Max Local Min Horizontal Point of Inflection

4. (16 points)
Your demand curve for selling x Items is given by

$$p = h(x) = 108 - x^2,$$

where quantity, x, is measured in Items and price, p, is measured in dollars. The graph of the demand curve is given at right. The demand curve is positive and decreasing from $x = 0$ to $x = 10.39$.



(a) Give the formula for the **Total Revenue** and **Marginal Revenue**.

ANSWER: $TR(x) =$ _____

$MR(x) =$ _____

(b) Find the quantity and price that corresponds to the largest possible value of total revenue over the interval $x = 0$ to $x = 10$.

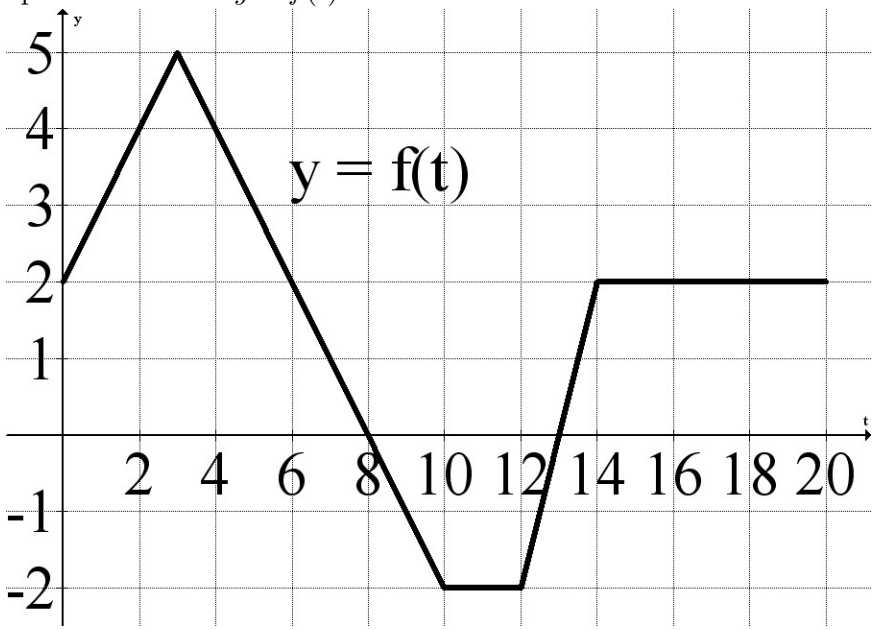
ANSWER: quantity = _____ Items

price = _____ dollars

(c) If you are given that the market equilibrium occurs at the quantity $x = 3$ items, find the consumer's surplus. (Recall: $CS = \int_0^{x_1} h(x)\,dx - p_1 x_1$)

ANSWER: consumer's surplus = _____ dollars

5. (16 pts) The graph of a function $y = f(t)$ is below.



Using the graph above, we define a new function

$$A(m) = \int_0^m f(t)\,dt$$

(a) Compute the following:

- $A'(6)$

ANSWER: $A'(6) =$ _____

- $A''(6)$

ANSWER: $A''(6) =$ _____

(b) ~~Compute the value of~~ $\int_8^{20} f(t)\,dt$

~~ANSWER:~~ $\int_8^{20} f(t)\,dt =$ _____

~~(c) Find all values of m between 0 and 20 at which $A(m)$ has a local minimum.~~

~~ANSWER: $m =$~~ _____

~~(d) Find the largest value of $A(m)$ on the interval $m = 2$ to $m = 10$.~~

~~Not covered~~ winter 2025

~~ANSWER: 'Max output from $A(m)$' =~~ _____

6. (17 pts) Your company produces and sells gloves and hats. In a given month, let x be in **hundreds of gloves** produced and sold and let y be in **hundreds of hats** produced and sold. The profit for the month in **hundreds of dollars** is given by:

$$P(x, y) = 8x + 4xy - 5x^2 - y^2 - 4 \text{ hundred dollars.}$$

(a) Compute the partial derivatives of P.

$P_x(x, y) =$ _____

$P_y(x, y) =$ _____

(b) Use a partial derivative to approximate the value of $\dfrac{P(3.0001, 4) - P(3, 4)}{0.0001}$.

ANSWER: $\frac{P(3.0001,4)-P(3,4)}{0.0001} \approx$ _____

(c) Consider the one variable function when we substitute $x = 1$, that is, consider $f(y) = P(1, y)$. Find the maximum and minimum value of this one variable function on the interval $y = 0$ to $y = 3$.

ANSWER: Max output value = _____

Min output value = _____

(d) You are told that the maximum of profit occurs at the critical point. Find the critical point of profit and give the maximum profit value.

Critical point: $(x, y) =$ _____

Maximum profit = _____ hundred dollars

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see last page" below a problem.