1. (12 points)

(a) Find the derivative of $f(x) = \frac{6}{\sqrt{x^3}} + 10\ln(x^4+1)$ at $x = 1$.

VERSION B
"20"
$\Rightarrow -9 + 20 \cdot \frac{1}{2} \cdot 4$
$\Rightarrow -9 + 40 = \boxed{31}$

$f(x) = 6x^{-3/2} + 10\ln(x^4+1)$

$f'(x) = 6\left(-\frac{3}{2}\right)x^{-5/2} + 10 \cdot \frac{1}{x^4+1} \cdot (4x^3)$

$f'(1) = \overbrace{-9} + 10 \cdot \frac{1}{2} \cdot 4$

$= -9 + 20 = 11$

$f'(1) =$ 11

(b) Evaluate the integral: $\int 3e^{5x} - \frac{x^4}{2} + \frac{5}{7x}\,dx$

VERSION B
"8"

$\int 3e^{5x} - \frac{1}{2}x^4 + \frac{5}{7}\frac{1}{x}\,dx$

$= \frac{3}{5}e^{5x} - \frac{1}{10}x^5 + \frac{5}{7}\ln(x) + C$

Answer = $\frac{3}{5}e^{5x} - \frac{1}{10}x^5 + \frac{5}{7}\ln(x) + C$

(c) Evaluate the integral: $\int_1^2 4x(x^2+1)\,dx$

VERSION B
"8"
FINAL ANSWER = 42

$\int_1^2 4x^3 + 4x\,dx$

$= x^4 + 2x^2 \Big|_1^2$

$= ((2)^4 + 2(2)^2) - ((1)^4 + 2(1)^2)$

$= (16 + 8) - 3$

$= 24 - 3$

Answer = 21

2. (15 points)

VERSION B SAME

(a) (5 pts) Find and classify all critical values for the function $f(x) = \frac{1}{3}x^3 + x^2 - 8x + 30$. (*Clearly, label if each critical value is a local max, local min or horizontal point of inflection*).

$f'(x) = x^2 + 2x - 8 \stackrel{?}{=} 0$

$(x+4)(x-2) = 0$

$x = -4, \ x = 2$

$\nearrow \quad \searrow \quad \nearrow$

$f' > 0 \quad -4 \quad f' < 0 \quad 2 \quad f' > 0$

ALSO $f''(x) = 2x + 2$

$f''(-4) = -14$ CONCAVE DOWN

$f''(2) = 6$ CONCAVE UP

LOCAL MAX LOCAL MIN

(List Critical values) $x =$ -4, 2

(b) Water is flowing into and out of two vats, Vat A and Vat B. You are given

- Vat A rate of flow: $A'(t) = 8 - 4t$ in gallons/hour.
- Vat B amount: $B(t) = -t^3 + 12t + 39$ in gallons. VERSION B " 29 "

Also, both vats have the same amount of water at time $t = 1$ hour (*i.e.* $A(1) = B(1)$).

i. (3 pts) Find the formula, $A(t)$, for the amount of water in the vat A at time t. (Remember to solve for "$+C$")

$A(t) = \int 8 - 4t\,dt = 8t - 2t^2 + C \Rightarrow 8 - 2 + C = 50$ ← 40

$C = 44$ ← 34

$A(1) = B(1) = -1 + 12 + 39 = 50$

$A(t) =$ $8t - 2t^2 + 44$ gallons.

ii. (3 pts) The function $B(t) = -t^3 + 12t + 39$ has one point of inflection. Find this point of inflection and determine the interval on which the function $B(t)$ is concave down

$B'(t) = -3t^2 + 12$

$B''(t) = -6t \stackrel{?}{=} 0 \Rightarrow t = 0$

$\cup \quad \cap$

$B'' > 0 \quad 0 \quad B'' < 0$

VERSION B 29

point of inflection: $(x, y) =$ $(0, 39)$

interval when concave down: $0 < x < \infty$

iii. (4 pts) What is the largest and smallest amount of water (in gallons) in **Vat B** in the first 4 hours?

$B'(t) = -3t^2 + 12 \stackrel{?}{=} 0$

$t^2 = 4$

$t = \pm 2$

ONLY $t = 2$ IS BETWEEN 0 AND 4

$B(0) = 39$ ← 29 VERSION B

$B(2) = -(2)^3 + 12(2) + 39$

$= -8 + 24 + 39 = 55$ ← 45

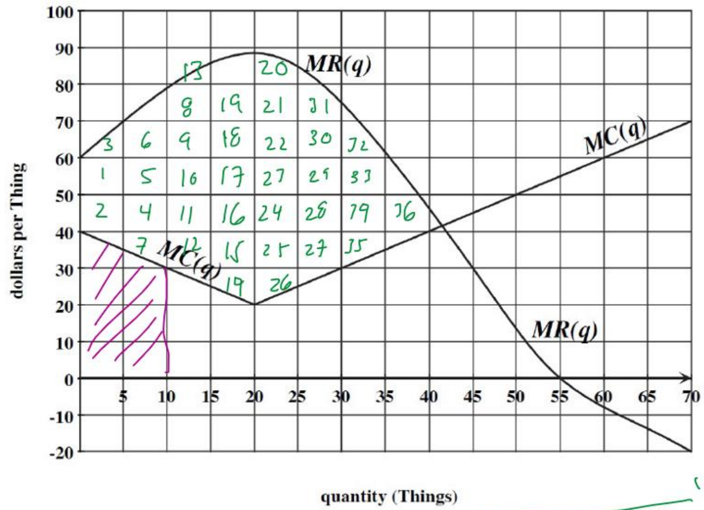
$B(4) = -(4)^3 + 12(4) + 39$

$= -64 + 48 + 39 = 23$ ← 13

global max = 55 gallons ← 45

global min = 23 gallons ← 13

3. (13 pts) Below are the graphs of marginal revenue and marginal cost for selling Things:



"100" VERSION B

250

Also assume that fixed cost is $FC = TC(0) = 25$ dollars.

(a) (6 pts) Estimate as accurately as possible from the graph:

i. $TC(10) =$ 375

AREA FROM 0 TO 10, FC

7 BOX · 50 = 350 + 25

ii. $TC'(10) =$ MC(10) = 30

iii. $TC''(10) =$ MC'(10) = slope = −1

(20,20) (0,40)

$\frac{40-20}{0-20} = \frac{+20}{-20} = -1$

(b) (3 pts) Give the quantity at which each of the following occur (estimate from the graph):

i. The graph of MR has a local maximum at $q =$ 20

ii. The graph of Profit has a local maximum at $q =$ ~42

iii. The graph of TR has a local maximum at $q =$ 55

SWAP VERSION B

(c) (4 pts) Estimate the maximum profit (in dollars).

AREA BETWEEN − FC ≃ 1850 − FC = 1825

ABOUT 36 TO 38 BOXES

37 BOXES

⇒ 37 · 50 = 1850

VERSION B ← 1750

max profit = 1825 dollars