Final Exam Overview

The final exam is comprehensive. Here is what I would do to study:

1. Finish the last several homework assignments early and make sure you know the new material (since exam 2) very well.

2. Go back and review exam 1 and exam 2 (along with the exam 1 and exam 2 review) to see if there are any particular topics you need to go back and look at again. You definitely should be very ready in case I ask a question just like one from a midterm (you don't want to be missing the same question twice in one quarter!).

3. If you find are weak in particular topics, then go look at the homework from those sections again.

4. Then you should be practicing on old finals. Treat these old finals like real exams! That is take the problems on your own and time yourself. Having a tutor help you through a final (or looking in your notes to help you with a final or working backward from the answers) does not count as studying. You need to be attempting the problem on your own.

The following pages contain a brief review of topics:

Foundations:

- Functional notation: Know how to work with a rule like $f(x) = x^2 - 3x$. What is $f(3)$? What is $f(x + h)$? You must know function basics.

- Types of Rates:

$$\text{“average rate of change of } f(x) \text{ from } a \text{ to } b\text{”} = \frac{f(b) - f(a)}{b - a}$$

$$\text{“average rate of change of } f(x) \text{ from } x \text{ to } x + h\text{”} = \frac{f(x + h) - f(x)}{h}$$

$$\text{“instantanteous rate of change of } f(x)\text{”} = f'(x) = \text{ “what } \tfrac{f(x+h)-f(x)}{h} \text{ becomes when } h \to 0\text{”}$$

- Slopes:

$$\text{“slope of the secant line to } f(x) \text{ from } x \text{ to } x + h\text{”} = \frac{f(x + h) - f(x)}{h}$$

$$\text{“slope of the tangent line to } f(x)\text{”} = f'(x)$$

- Concerning the rates and slopes above, you should be able to work from any given information. Meaning if we give you $f(x)$, you should be able to find $\frac{f(x+h)-f(x)}{h}$ and $f'(x)$. And if we instead give you $\frac{f(x+h)-f(x)}{h}$ then you should be able to find $f'(x)$ and information about $f(x)$. See an example of such a problem on exam 1.

- Be able to find the equation of a tangent line to $f(x)$ at $x = a$. Remember the tangent line equation is given by

$$\text{Equation for tangent line to } f(x) \text{ at } x = a : \; y = f'(a)(x - a) + f(a)$$

- Be able to estimates slope and rate information directly from the graph of a function.

Derivatives and Antiderivatives:
You have had so many practice problems with derivatives and antiderivatives. There is really no good excuse for not being proficient with basic derivatives and antiderivatives at this point.

If you are still making mistakes, then you really need to go back and read my review sheets. My 9.3/9.4, 9.5/9.6, 9.7/9.8 and 11.1/11.2 reviews all have worked out and fully explained derivative examples. And my 12.1/12.3 review sheet contains every type of integral you could see on the final with worked out solutions. So no integral or derivative should be able to surprise you! And you shouldn't even make any small mistakes since we have done these so many times (and we often have ways to check our work). So make sure you have these down well.

Here are those **derivative** rules again:

NAME	RULE	EXAMPLE
Power Rule	$(x^n)' = nx^{n-1}$	$\left(\frac{3}{2x^5}\right)' = \left(\frac{3}{2}x^{-5}\right)' = -\frac{15}{2}x^{-6}$
Sum Rule	$(f+g)' = f' + g'$	$(e^{2x} + \ln(x))' = e^{2x} \cdot 2 + \frac{1}{x}$
Coefficient Rule	$(cf)' = cf'$	$(10e^{4x})' = 10e^{4x} \cdot 4$
Product Rule	$(f \cdot g)' = f \cdot g' + g \cdot f'$	$(x^2 \ln(x))' = x^2 \cdot \frac{1}{x} + \ln(x) \cdot 2x$
Quotient Rule	$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{(g)^2}$	$\left(\frac{e^{4x}}{\sqrt{x}}\right)' = \frac{x^{1/2}e^{4x}4 - e^{4x}\frac{1}{2}x^{-1/2}}{x}$
General Power Rule	$([f(x)]^n)' = n[f(x)]^{n-1} \cdot f'(x)$	$((x^4 - x^2)^{30})' = 30(x^4 - x^2)^{29} \cdot (4x^3 - 2x)$
General Exponential Rule	$\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)$	$\left(e^{x^3+2x}\right)' = e^{x^3+2x} \cdot (3x^2 + 2)$
General Logarithm Rule	$(\ln(f(x)))' = \frac{1}{f(x)} \cdot f'(x)$	$(\ln(3x^5 + e^x))' = \frac{1}{3x^5+e^x} \cdot (15x^4 + e^x)$

Here are those **antiderivative** rules again (along with the sum and coefficient rules):

NAME	RULE	EXAMPLE
Constant Rule	$\int k\,dx = kx + C$	$\int 3\,dx = 3x + C$
Power Rule	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$	$\int \frac{x^4}{3} - \frac{5}{3x^7} + \sqrt{x}\,dx = \frac{1}{15}x^5 + \frac{5}{18}x^{-6} + \frac{2}{3}x^{3/2} + C$
Logarithm Rule	$\int \frac{1}{x}\,dx = \ln(x) + C$	$\int \frac{7x^5 - x}{x^2}\,dx = \int 7x^3 - \frac{1}{x}\,dx = \frac{7}{4}x^4 - \ln(x) + C$
Exponential Rule	$\int e^{nx}\,dx = \frac{1}{n}e^{nx} + C$	$\int e^{3x} + \frac{2}{3} - \frac{x}{6}\,dx = \frac{1}{3}e^{3x} + \frac{2}{3}x - \frac{1}{12}x^2 + C$

You should also know how to find the particular value of C when given some initial information about the antiderivative.

Areas and Definite Integrals:
Know how to compute definite integrals. Here is the brief review (from just last week) about what we did in chapter 13:

- $\int f(x)\,dx$ = indefinite integral = general antiderivative of $f(x)$ (will include a $+C$)
- $\int_a^b f(x)\,dx$ = definite integral = signed area between $f(x)$ and x axis from $x = a$ to $x = b$ (this will be a number).
- $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is any antiderivative of $f(x)$ (that means $F'(x) = f(x)$).
  This is the fundamental theorem of calculus.
  Notable business uses of the fundamental theorem:
  - $TR(x) = \int_0^x MR(q)\,dq$
  - $VC(x) = \int_0^x MC(q)\,dq$
  - $TC(x) - TC(0) = \int_0^x MC(q)\,dq$, so $TC(x) = \int_0^x MC(q)\,dq + FC$
  - $P(x) = \int_0^x MR(x)\,dx - \int_0^x MC(x)\,dx - FC = \int_0^x MR(x) - MC(x)\,dx - FC$
  - If $r(t)$ is the rate of income flow (in other words it is the derivative of the income formula) in dollars/years, then
    "Total income from $t = 0$ to $t = a$" $= \int_0^a r(t)\,dt$.
- If $f(x)$ is above $g(x)$ from $x = a$ to $x = b$, then the area between $f(x)$ and $g(x)$ from $x = a$ to $x = b$ is given by
  Area between $= \int_a^b f(x) - g(x)\,dx$.
- Suppose $p = f(x)$ is the demand function and $p = g(x)$ is the supply function. If market equilibrium occurs at $x = x_1$ and $p = p_1$ (you find this by getting the x and y coordinates of the intersection from $f(x) = g(x)$), then
  Consumer Surplus $= \int_0^{x_1} f(x)\,dx - p_1x_1$
  Producer Surplus $= p_1x_1 - \int_0^{x_1} g(x)\,dx$.

Fundamental Analysis Tools:
This quarter we learned how to analyze functions by using derivatives and antiderivatives. All of these methods ultimate come down to knowing the basic connections below:

ORIGINAL ($f(x)$)	DERIVATIVE ($f'(x)$)	SECOND DERIVATIVE ($f''(x)$)
$f(b) - f(a) = \int_a^b f'(x)\,dx$ $=$ Area under $f'(x)$	$f'(x) =$ slope on $f(x)$	$f''(x) =$ slope on $f'(x)$
increasing (uphill left-to-right)	positive (above x-axis)	
decreasing (downhill left-to-right)	negative (below x-axis)	
horizontal tangent	zero (crosses x-axis)	
concave up	increasing	positive
concave down	decreasing	negative
possible inflection point	horizontal	zero

As discussed in class, you always need to:

1. Know what is given.
2. Know what you want.
3. Use the appropriate connections.

For example, if you are given $f(x)$ and you want:

1. to find when $f(x)$ is positive or negative or zero, you need to first solve $f(x) = 0$.
2. to find when $f(x)$ is increasing or decreasing or horizontal, you need to first solve $f'(x) = 0$.
3. to find when $f(x)$ is concave up or down or has an inflection point, you need to first solve $f''(x) = 0$.

In addition, if $A(m) = \displaystyle\int_0^m f(x)dx$ and you are given information (or a graph) about $f(x)$, then you would know that

1. $A(m) =$ “the area under $f(x)$ from 0 to m.
2. $A'(x) = f(x) =$ “the height of the graph $f(x)$ at x”
3. $A''(x) = f'(x) =$ “the slope of the graph $f(x)$ at x”

The above facts are all you need to answer any question about local max/min, global max/min, increasing/decreasing, concave up/down, or inflection points. But if it helps you can review step-by-step instructions for all these individual methods if you read my 10.1-10.3 review sheets (several visual examples are given) and there is a separate review sheet about global max and min.

Applications and Comparing Derivatives:

1. Know when and how to do **derivatives** in applications when going from TR to MR, TC/VC to MC, P to MP, amount of water in a vat to rate of flow, height to rate of ascent, dist to speed, or total amount to rate of change of amount.

2. Know when and how to do **antiderivatives** in applications when going from MR to TR, MC to TC/VC, MP to P, rate of flow to amount of water, rate of ascent to height, speed to dist, or any rate of change to total amount.

3. Know how to look at a graph and find areas in order to make conclusions about the changes in the antiderivative.

When looking at two derivative graphs (MR and MC, or the speeds for two cars, or the rates of ascent for two Balloons, or the rate of flow for two vats), we should know that critical things are happening when the derivatives are equal! Here is a summary of many of these situations we have seen:

- Profit is maximize at a quantity where $MR = MC$ (we want the intersection where it switches from $MR > MC$ before to $MR < MC$ after). The **area between** MR and MC is the change in profit.

- To find when Balloon A is farthest above Balloon B, we want to find a time when $A'(t) = B'(t)$ (we want the intersection where it switches from $A' > B'$ before to $A' < B'$ after). In these situations you should be able to look at the information from the beginning and know which balloon was higher after the start (often that is a useful thing to know). The **area between** $A'(t)$ and $B'(t)$ is the change in distance between them.

Multivariable Calculus and Partial Derivatives
I am not going to say to much about this topic in this review sheet since we just covered this material. You should see my chapter 14 review sheet for a deeper overview. Here is a very brief summary.

1. Know how to compute partial derivatives.
2. Be able to interpret partial derivatives
3. Know how to find critical points.

Essential algebra skills

Here are the algebra skills you used the most often in the homework.

1. Simplifying and rewriting powers before doing a derivative or integral. Here are several examples taken directly from homework (this was the first step before you could take a derivative or find an antiderivative in these problems):

SIMPLIFYING/REWRITING ALGEBRA YOU NEEDED
$-\frac{6}{7x} = -\frac{6}{7}x^{-1}$, $\frac{5}{e^{3x}} + \frac{e^{8x}}{2} = 5e^{-3x} + \frac{1}{2}e^{8x}$
$\sqrt{\ln(3x+5)} = (\ln(3x+5))^{1/2}$, $\frac{3x}{400} = \frac{3}{400}x$
$8\sqrt{x} = 8x^{1/2}$, $\frac{5}{3x^6} = \frac{5}{3}x^{-6}$, $\frac{6}{7\sqrt{x^3}} = \frac{6}{7}x^{-3/2}$, $\frac{7}{x^8} = 7x^{-8}$ $(5x^2-4)^2x^3 = (25x^4 - 40x^2 + 16)x^3 = 25x^7 - 40x^5 + 16x^3$ $\frac{x+7}{x^6} = \frac{x}{x^6} + \frac{7}{x^6} = x^{-5} + 7x^{-6}$ $\frac{4}{e^{9x}} = 4e^{-9x}$, $\frac{3}{e^{x/2}} = 3e^{-\frac{1}{2}x}$

2. Solving equations comes up when you have to find critical numbers. I went through all your homework and here is summary of what you had to do in terms of solving equations. Go back to the course website and you will also see a full review of all the solving techniques you need in this class.

HOMEWORK	SOLVING EQUATIONS ALGEBRA YOU NEEDED
10.1	You had one linear equation to solve and six quadratic equations to solve. Remember it is worth trying to factor first, for example to solve: $x^3 - 3x^2 = 0$, we can factor to get $x^2(x-3) = 0$, so $x = 0$ and $x = 3$ are the solutions.
10.2	You had to solve mostly quadratic equations and one that had fractions. If an equation has fractions first clear the denominators. For example, $\frac{-9}{(t+1)^2} + \frac{144}{(t+1)^3} = 0$ becomes $-9(t+1) + 144 = 0$
10.3	You had quite a few linear and quadratic equations again. Here was one with a fraction: $2 + \frac{-4}{(3x-1)^2} = 0$ became $2(3x-1)^2 - 4 = 0$.
12.4	You had to solve an equation with a root. $60\sqrt{q+4} = 1200$ becomes $\sqrt{q+4} = 20$ which becomes $q + 4 = 400$, so $q = 396$.

3. Solving systems of equations to find critical values for multivariable functions. This comes up in the multivariable sections (namely 14.3). Read my chapter 14 review sheet for examples.