Exam 2 Overview

Exam 2 covers 10.1-10.3, 11.1, 11.2, 12.1, 12.3, 12.4, 13.2, and 13.3. Essentially it is about all the derivatives, derivative applications, and the basic facts of indefinite and definite integrals. See my Exam 1 Review for studying advice along with the old emails about studying and my general studying advice sheet all on the course website.

Know your basic derivative and anti-derivative skills (You had to do this a lot in 11.1/11.2, 12.1/12.3, 12.4, 13.2 and 13.3):

1. We added the two derivatives $(e^{f(x)})' = e^{f(x)} \cdot f'(x)$ and $(\ln(f(x)))' = \frac{1}{f(x)} \cdot f'(x)$. You should be very comfortable with taking the derivatives of even very crazy functions by just using combinations of these two rules with all our old rules. See the 11.1/11.2 homework and the review problems in the exam archive for more practice.

2. We learned antiderivatives, there are only 4 rules:

Constant Rule	$\int a\,dx = ax + C$	$\int 3\,dx = 3x + C$
Power Rule	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$	$\int x^5\,dx = \frac{1}{6}x^6 + C$ (works for $n \neq -1$)
Logarithm Rule	$\int \frac{1}{x}\,dx = \ln(x) + C$	(works for positive x)
Exponential Rule	$\int e^{nx}\,dx = \frac{1}{n}e^{nx} + C$	$\int e^{-5x}\,dx = -\frac{1}{5}e^{-5x} + C$

   All integrals in this class can be done by simplifying then using the four rules above.

3. Be able to compute definite integrals with the fundamental theorem of calculus. Here are one example to remind you (see your homework, my 13.2 review sheet, and old exams for more practice):
   $\int_2^5 3x^2 + 2\,dx = x^3 + 2x\Big|_2^5 = [(5)^3 + 2(5)] - [(2)^3 + 2(2)] = 135 - 12 = 123.$

Know how to analyze a function using derivatives (You had to do this in 10.1-10.3, then again in 12.4 and 13.2 and 13.3):

1. Know how to find: critical numbers, critical points, local max/min, global max/min, intervals when f(x) is increasing/decreasing, intervals when f(x) is concave up/down, points of inflections and horizontal points of inflection.

2. Here's a reminder of the basic processes:

   - If a question asks about **critical numbers, critical points, local max/min, or increasing/decreasing**, then

     (a) *What is the 'original' function?* (Is the question asking about distance or speed or MR or TC or what?). Think of this function as $f(x)$.

     (b) *Critical Points.* Find $f'(x)$ and solve $f'(x) = 0$ for x.

     (c) *1st or 2nd Derivative Test, your choice.* Use the 1st or 2nd derivative test, here's a reminder about how those work:
     1st Derivative Test: Draw a number line with the critical numbers labeled as tick marks. Between each tick mark figure out if $f'(x)$ is positive or negative. Make appropriate conclusions about whether $f(x)$ is increasing or decreasing.
     2nd Derivative Test: Find $f''(x)$. Plug each critical number into $f''(x)$. If $f''(x)$ is positive, then the function is concave up at the critical number (local min). If $f''(x)$ is negative, then the function is concave down at the critical number (local max).

   - If a question asks about **global max/min** on an interval, then

     (a) *What is the 'original' function?* Think of this function as $f(x)$.

     (b) *Critical Points.* Find $f'(x)$ and solve $f'(x) = 0$ for x.

     (c) *See which has the biggest/smallest original output.* Plug all the critical points **and** the endpoints of the interval into $f(x)$ and see which gives the biggest and smallest outputs.

   - If a question asks about **points of inflection or concave up/down**, then

     (a) *What is the 'original' function?* Think of this function as $f(x)$.

     (b) *Possible Points of Inflection.* Find $f''(x)$ and solve $f''(x) = 0$ for x.

     (c) *2nd Deriv Number Line.* Draw a number line with the possible points of inflection labeled as tick marks. Between each tick mark figure out if $f''(x)$ is positive or negative. Make appropriate conclusions about whether $f(x)$ is concave up or concave down.

All of these ideas come back to the same connections we have been talking about for several weeks now. Here is that table once more with a few more connections filled in:

ORIGINAL ($f(x)$)	DERIVATIVE ($f'(x)$)	SECOND DERIVATIVE ($f''(x)$)
$f(b) - f(a) = \int_a^b f'(x)\,dx$ $=$ Area under $f'(x)$	$f'(x) =$ slope on $f(x)$	$f''(x) =$ slope on $f'(x)$
increasing (uphill left-to-right)	positive (above x-axis)	
decreasing (downhill left-to-right)	negative (below x-axis)	
horizontal tangent	zero (crosses x-axis)	
concave up	increasing	positive
concave down	decreasing	negative
possible inflection point	horizontal	zero

Be able to interpret our results and use them in applications:

1. Know when and how to do **derivatives** in applications when going from TR to MR, TC/VC to MC, P to MP, amount of water in a vat to rate of flow, height to rate of ascent, dist to speed, or total amount to rate of change of amount.

2. Know when and how to do **antiderivatives** in applications when going from MR to TR, MC to TC/VC, MP to P, rate of flow to amount of water, rate of ascent to height, speed to dist, or any rate of change to total amount.

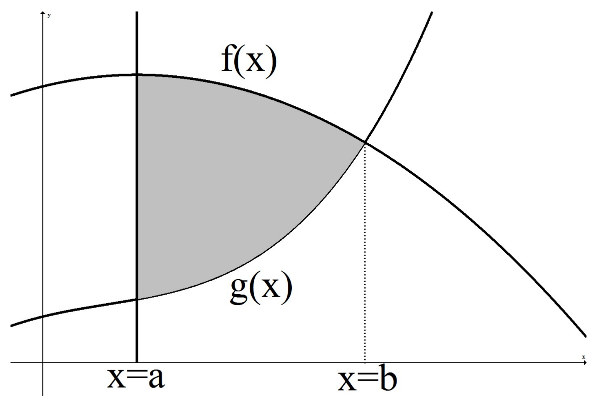
3. In the case of general antiderivative formulas, know how to use initial conditions to find the constant of integration C.

4. Know how to look at a graph and find areas in order to make conclusions about the changes in the antiderivative.

Be able to interpret areas under and between curves like we did in 13.2/13.3

We noted that:

$$\text{Area between curves } = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b f(x) - g(x)\,dx$$



Applications of areas:

$TR(x) - TR(0) = \int_0^x MR(q)\,dq =$ 'area between MR(q) and the q-axis from 0 to x'.

$TC(x) - TC(0) = \int_0^x MC(q)\,dq =$ 'area between MC(q) and the q-axis from 0 to x'.

$P(x) - P(0) = \int_0^x MR(q) - MC(q)\,dq =$ 'area between $MR(q)$ and $MC(q)$ from 0 to x'.

(in this last one, we treat the area as positive if $MR > MC$ and negative if $MR < MC$).

Also note that we always know that:

$$TR(0) = 0$$
$$TC(0) = FC = \text{ "Fixed Cost"}$$
$$P(0) = -FC$$

(so you must remember to add or subtract fixed cost appropriately when computing areas and trying to get TC or Profit).

Essential algebra skills

Here are the algebra skills you used the most often in the homework.

1. Simplifying and rewriting powers before doing a derivative or integral. Here are several examples taken directly from homework (this was the first step before you could take a derivative or find an antiderivative in these problems):

HOMEWORK	SIMPLIFYING/REWRITING ALGEBRA YOU NEEDED
11.1/11.2 (Part 1)	$-\frac{6}{x} = -6x^{-1}$, $\frac{5}{e^{3x}} + \frac{e^{8x}}{2} = 5e^{-3x} + \frac{1}{2}e^{8x}$
11.1/11.2 (Part 2)	$\sqrt{\ln(3x+5)} = (\ln(3x+5))^{1/2}$, $\frac{x}{400} = \frac{1}{400}x$
12.1/12.3	$8\sqrt{x} = 8x^{1/2}$, $\frac{5}{x^6} = 5x^{-6}$, $\frac{6}{7\sqrt{x^3}} = \frac{6}{7}x^{-3/2}$, $\frac{7}{x^8} = 7x^{-8}$ $(5x^2-4)^2x^3 = (25x^4 - 40x^2 + 16)x^3 = 25x^7 - 40x^5 + 16x^3$ $\frac{x+1}{x^6} = \frac{x}{x^6} + \frac{1}{x^6} = x^{-5} + x^{-6}$ $\frac{4}{e^{9x}} = 4e^{-9x}$, $\frac{3}{e^{x/2}} = 3e^{-\frac{1}{2}x}$

2. Solving equations comes up when you have to find critical numbers. I went through all your homework and here is summary of what you had to do in terms of solving equations. Go back to the course website and you will also see a full review of all the solving techniques you need in this class.

HOMEWORK	SOLVING EQUATIONS ALGEBRA YOU NEEDED
10.1	You had one linear equation to solve and six quadratic equations to solve. Remember it is worth trying to factor first, for example to solve: $x^3 - 3x^2 = 0$, we can factor to get $x^2(x-3) = 0$, so $x = 0$ and $x = 3$ are the solutions.
10.2	You had to solve mostly quadratic equations and one that had fractions. If an equation has fractions first clear the denominators. For example, $\frac{-9}{(t+1)^2} + \frac{144}{(t+1)^3} = 0$ becomes $-9(t+1) + 144 = 0$
10.3	You had quite a few linear and quadratic equations again. Here was one with a fraction: $2 + \frac{-4}{(3x-1)^2} = 0$ became $2(3x-1)^2 - 4 = 0$.
12.4	You had to solve an equation with a root. $60\sqrt{q+4} = 1200$ becomes $\sqrt{q+4} = 20$ which becomes $q + 4 = 400$, so $q = 396$.