18 16

1. (16 pts) For each of the following, you do not need to simplify.

(a) Find the derivative of $f(x) = \frac{3}{2x^4} - \frac{10}{\sqrt{x}} - \frac{5x}{3}$. $= \frac{3}{2} x^{-4} - 10 x^{-\frac{1}{2}} - \frac{5}{3} x$

$f'(x) = -6x^{-5} + 5x^{-3/2} - 5/3$

4

VERSION B "3" → "7"

"$\frac{5}{3}$" → "$\frac{7}{4}$"

ANSWER: $f'(x) =$ $-\frac{6}{x^5} + 5x^{-3/2} - \frac{5}{3}$

(b) Find the 1st and 2nd derivatives of $g(x) = \ln(x^3 + 4)$.

VERSION B "4" → "5"

3

ANSWER: $g'(x) =$ $\frac{3x^2}{x^3+4}$

$N = 3x^2 \qquad D = x^3 + 4$

$N' = 6x \qquad D' = 3x^2$

$\frac{DN' - D'N}{D^2} = \frac{(x^3+4)(3x^2) - 3x^2(3x^2)}{(x^3+4)^2}$

3

ANSWER: $g''(x) =$ ______

VERSION B "4" → "3"

"3" → "2"

(c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = \frac{4x}{y} + 5xy^4 - e^{3y} + \ln(x)$. $= 4xy^{-1} + 5xy^4 - e^{3y} + \ln(x)$

6

ANSWERS: $\frac{\partial z}{\partial x} =$ $\frac{4}{y} + 5y^4 + \frac{1}{x}$ $\qquad \frac{\partial z}{\partial y} =$ $-4xy^{-2} + 20xy^3 - 3e^{3y}$

2. (~~15~~ 17 pts)

(a) Find the equation for the tangent line to $f(x) = 5xe^{3x} + \sqrt{2x+4}$ at $x = 0$.
Reminder: Your answer will be a line, *i.e.* $y = mx + b$. So you need to find the slope and the y-intercept of this line.

$f'(x) = 5e^{3x} + 15xe^{3x} + \frac{1}{2}(2x+4)^{-1/2} \cdot 2$

$f(0) = 0 + \sqrt{4} = 2$

$f'(0) = 5 + 0 + \frac{1}{2}\frac{1}{\sqrt{4}} \cdot 2 = 5 + \frac{1}{2} = 5.5$

ANSWER: $y = 5.5x + 2$

5

VERSION B : "5" → "2"

(b) Find $g'(x)$ if $\dfrac{g(x+h) - g(x)}{h} = \dfrac{6}{x+h} - 4x + x^2h$

LET $h \to 0 \Rightarrow g'(x) = \frac{6}{x+0} - 4x + x^2 \cdot 0$

ANSWER: $g'(x) = \frac{6}{x} - 4x$

4

(c) Evaluate the integrals:

i. $\displaystyle\int \left(2e^{-3x} + \frac{x^2 - 5x}{x^2}\right) dx$

$\int 2e^{-3x} + 1 - 5\frac{1}{x}\,dx$

$= \frac{2}{-3}e^{-3x} + x - 5\ln|x| + C$

ANSWER: $-\frac{2}{3}e^{-3x} + x - 5\ln|x| + C$

4

NO CHANGE

ii. $\displaystyle\int_1^3 \left(\frac{18}{x^2} + 2x\right) dx$

$\int_1^3 18x^{-2} + 2x\,dx = -18x^{-1} + x^2\Big|_1^3$

$= (-18(\frac{1}{3}) + 9) - (-18 + 1)$

$= -6 + 9 - (-17) = -6 + 9 + 17$

$= 20$

ANSWER: 20

4

3. (18 points)

(a) The formulas for two functions are:

$$f(x) = x^2 - 8x + 12 \quad \text{and} \quad g(x) = \frac{4}{3}x^3 - 26x^2 + 88x + 600.$$

i. Find the global maximum and global minimum values of $g(x)$ over the interval $x = 0$ to $x = 8$.

$$g'(x) = 4x^2 - 52x + 88 = 4(x^2 - 13x + 22) = 4(x-2)(x-11) = 0$$

$$@\ 2, \cancel{11}$$

6

$g(0) = 600$

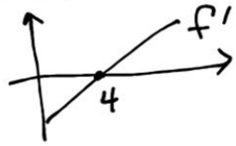
$g(2) = 682.67$

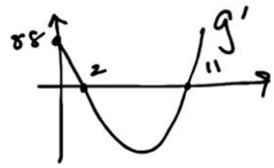
$g(8) = 322.67$

ANSWERS: MIN VALUE = 322.67 MAX VALUE = 682.67

ii. Find the longest interval on which $f(x)$ is decreasing and $g(x)$ is decreasing.

$f'(x) = 2x - 8 = 0 @ x = 4$





6

f decreasing for $x < 4$

g decreasing for $2 < x < 11$

ANSWER: $x =$ 2 to $x =$ 4

(b) The function $h(x) = 12\ln(x) - 2x + 6$ has one critical number, find it and identify whether it corresponds to a local max, a local min, or a horizontal point of inflection.

$h'(x) = \frac{12}{x} - 2 = 0$

$\frac{12}{x} = 2$

$x = 6$

$h''(x) = -\frac{12}{x^2} < 0$ ⌒

local max

6

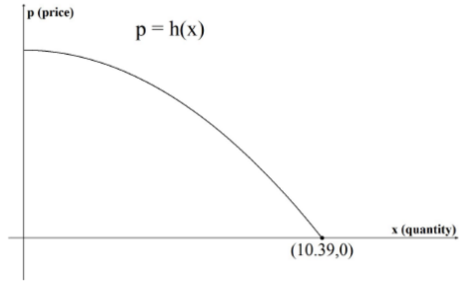
ANSWER: $x =$ 6

Circle one: Local Max Local Min Horizontal Point of Inflection

16

4. (12 points)
Your demand curve for selling x Items is given by

$$p = h(x) = 108 - x^2,$$



where quantity, x, is measured in Items and price, p, is measured in dollars. The graph of the demand curve is given at right. The demand curve is positive and decreasing from $x = 0$ to $x = 10.39$.

(a) (~~2 points~~) Give the formula for the **Total Revenue** and **Marginal Revenue**.
4

ANSWER: $TR(x) =$ $108x - x^3$ 2

$MR(x) =$ $108 - 3x^2$ 2

(b) (~~2 points~~) Find the quantity and price that corresponds to the largest possible value of total revenue over the interval $x = 0$ to $x = 10$. ←

$108 - 3x^2 = 0 \Rightarrow x^2 = \frac{108}{3} = 36 \Rightarrow x = 6$

$TR(6) = 108(6) - (6)^3 = 432 \leftarrow$ MAX 6

$TR(0) = 0$

$TR(10) = 108(10) - (10)^3 = 80$

$p = h(6) = 108 - (6)^2 = 72$

ANSWER: quantity = 6 Items

price = 72 dollars

(c) (~~1 pts~~) If you are given that the market equilibrium occurs when the quantity is 3 items dollars, find the consumer's surplus. (Recall: $CS = \int_0^{x_1} h(x)\,dx - p_1 x_1$)

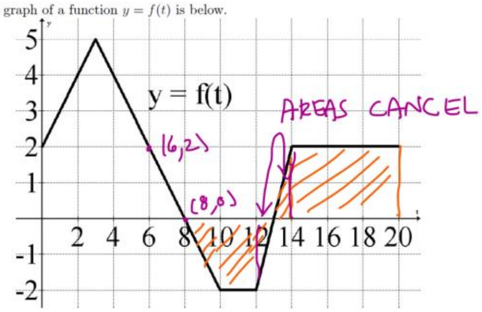
$x = 3 \Rightarrow p = 108 - 3^2 = 99$

$\underbrace{\int_0^3 108 - x^2\,dx}_{108x - \frac{1}{3}x^3 \big|_0^3} - 99 \cdot 3$ 6

$108x - \frac{1}{3}x^3 \Big|_0^3 - 297 = 108(3) - \frac{1}{3}(3)^3 - 297$
$= 18$

ANSWER: consumer's surplus = 18 dollars

5. (16 pts) The graph of a function $y = f(t)$ is below.



Using the graph above, we define a new function

$$A(m) = \int_0^m f(t)\,dt$$

(a) Compute the following:

- $A'(6) = f(6) = 2$

ANSWER: $A'(6) =$ 2

- $A''(6) = f'(6) = \text{slope} = \frac{2-0}{6-8} = \frac{2}{-2} = -1$

ANSWER: $A''(6) =$ −1

(b) Compute the value of $\int_8^{20} f(t)\,dt$ $= -2 - 4 + 0 + 6 \cdot 2 = -6 + 12$

AREA $= \frac{1}{2}(2)(2) = -2$ AREA $= -4$

ANSWER: $\int_8^{20} f(t)\,dt =$ 6

(c) Find all values of m between 0 and 20 at which $A(m)$ has a local minimum.

A' CHANGES FROM − TO +

ANSWER: $m =$ 13

(d) Find the global maximum value of $A(m)$. from $m=2$ to $m=10$.

OCCURS AT $m = 8$

VALUE $=$
$\frac{1}{2}(3)(5+2) + \frac{1}{2}5(5)$

$\frac{21}{2} + \frac{25}{2}$

ANSWER: 'Max output from $A(m)$' $=$ 23

3

3

3

3

4

VERSION B

$A'(2) = 4$

$A''(2) = 1 = \frac{5-2}{3-0} = \frac{3}{3} = 1$

17

6. (16 pts) Your company produces and sells gloves and hats. In a given month, let x be in **hundreds of gloves** produced and sold and let y be in **hundreds of hats** produced and sold. The profit for the month in **hundreds of dollars** is given by:

$$P(x, y) = 8x + 4xy - 5x^2 - y^2 - 4 \text{ hundred dollars.}$$

(a) Compute the partial derivatives of P.

$P_x(x, y) =$ $8 + 4y - 10x$

$P_y(x, y) =$ $4x - 2y$

4

(b) Use a partial derivative to approximate the value of $\dfrac{P(3.0001, 4) - P(3, 4)}{0.0001}$.

$P_x(3,4) = 8 + 4(4) - 10(3) = 8 + 16 - 30$

2

VERSION B
(3,2)
$\Rightarrow 8 + 4(2) - 10(3)$
$16 - 30 = -14$

ANSWER: $\frac{P(3.0001,4)-P(3,4)}{0.0001} \approx$ -6

(c) Consider the one variable function when we substitute $x = 1$, that is, consider $f(y) = P(1, y)$. Find the maximum and minimum value of this one variable function on the interval $y = 0$ to $y = 3$.

$f(y) = 8 + 4y - 5 - y^2 - 4 = -1 + 4y - y^2$

$f'(y) = 4 - 2y = 0 \Rightarrow y = 2$

$f(0) = -1$

$f(2) = -1 + 8 - 4 = 3$

$f(3) = -1 + 12 - 9 = 2$

5

ANSWER: Max output value = 3

Min output value = -1

(d) You are told that the maximum of profit occurs at the critical point. Find the critical point of profit and give the maximum profit value.

① $8 + 4y - 10x = 0$

② $4x - 2y = 0 \rightarrow y = 2x$

$8 + 4(2x) - 10x = 0$

$8 + 8x - 10x = 0$

$8 - 2x = 0$

$x = 4$

$y = 8$

6

Critical point: $(x, y) =$ $(4, 8)$

$P(4,8) = 32 + 16 \cdot 8 - 5(4)^2 - (8)^2 - 4$

Maximum profit = 12 hundred dollars