Quiz 2

"8" VERSION B — "114" — "12"

1. (10 pts) You sell items. The functions for marginal revenue and marginal cost (in dollars/item) are given by

$$MR(q) = 9e^{0.02q} \quad \text{and} \quad MC(q) = q^2 - 6q + 124,$$

where q is in thousands of items. You are also told that Fixed Costs are given $FC = 13$ thousand dollars (so $TC(0) = 13$).

(a) Give the functions for Total Revenue and Total Cost (solve for the constants of integration).

$TR(q) = \int 9e^{0.02q}\,dq = \frac{9}{0.02}e^{0.02q} + C = 450e^{0.02q} + C$

$TR(0) = 0 \Rightarrow 450e^0 + C = 0 \Rightarrow C = -450$

VERSION B

$TR(q) = 400e^{0.02q} - 400$

$TC(q) = \frac{1}{3}q^3 - 3q^2 + 114q + 12$

$TC(q) = \int q^2 - 6q + 124\,dq = \frac{1}{3}q^3 - 3q^2 + 124q + C$

$TC(0) = 13 \Rightarrow 0 - 0 + 0 + C = 13$
$\Rightarrow C = 13$

$TR(q) = \underline{\quad 450e^{0.02q} - 450 \quad}$

$TC(q) = \underline{\quad \frac{1}{3}q^3 - 3q^2 + 124q + 13 \quad}$

(b) Find the largest and smallest values of Marginal Cost on the interval $q = 0$ to $q = 10$.

$MC'(q) = 2q - 6 \overset{?}{=} 0 \Rightarrow q = 3$

VERSION B

$MC(0) = (0)^2 - 6(0) + 124 = 124$
$MC(3) = (3)^2 - 6(3) + 124 = 9 - 18 + 124 = 115$
$MC(10) = (10)^2 - 6(10) + 124 = 100 - 60 + 124 = 164$

VERSION B
$MC(0) = 114$
$MC(3) = 105$
$MC(10) = 154$

'smallest value of MC' = $\underline{\quad 115 \quad}$ dollars/item
'largest values of MC' = $\underline{\quad 164 \quad}$ dollars/item

(c) Recall: $AC(q) = \frac{TC(q)}{q}$.

Determine if $AC(q)$ is concave up, concave down, or neither at $q = 1$ thousand items. (In your work, you MUST find the second derivative of $AC(q)$ and make correct conclusions).

$AC(q) = \frac{1}{3}q^2 - 3q + 124 + 13q^{-1}$

VERSION B
"12" → "24"

$AC'(q) = \frac{2}{3}q - 3 - 13q^{-2}$

$AC''(q) = \frac{2}{3} + 26q^{-3} = \frac{2}{3} + \frac{26}{q^3}$

$AC''(1) = \frac{2}{3} + 26 > 0$

ANSWER: $AC''(q) = \underline{\quad \frac{2}{3} + \frac{26}{q^3} \quad}$

(Circle one) (CONCAVE UP)   CONCAVE DOWN   NEITHER