1. (16 points) Let $f(x) = 4x^3 - 78x^2 + 432x$.

   (a) Find all critical numbers of $f(x)$ and use the Second Derivative Test to determine whether each gives a local maximum or a local minimum value of $f(x)$.

   ANSWER: $x =$ ____________________ gives a local ____________________

   ANSWER: $x =$ ____________________ gives a local ____________________

   (b) Define a new function $D(x)$ by $D(x) = \dfrac{f(x)}{x}$. Find the value of $x$ at which $D(x)$ reaches its smallest value. (Your work should include an explanation of how you know $D(x)$ is smallest there.)

   ANSWER: $x =$ ____________________

   (c) Define a new function $S(x)$ by $S(x) = \dfrac{D(x)}{x}$. Find all positive critical numbers of $S(x)$.

   ANSWER: $x =$ ____________________

2. (19 points) You sell Gizmos. Your total revenue and total cost are given by the functions $TR(q) = -2q^2 + 199.1q$ and $TC(q) = 0.01q^3 - 2.405q^2 + 200q + 20$, where $q$ is in thousands of Gizmos and $TR$ and $TC$ are both in thousands of dollars.

(a) Find the largest interval on which $MR(q)$ is positive.

ANSWER: from $q =$____________to $q =$____________thousand Gizmos

(b) Is $TC(q)$ concave up or concave down at $q = 100$?

ANSWER: (circle one) concave up concave down

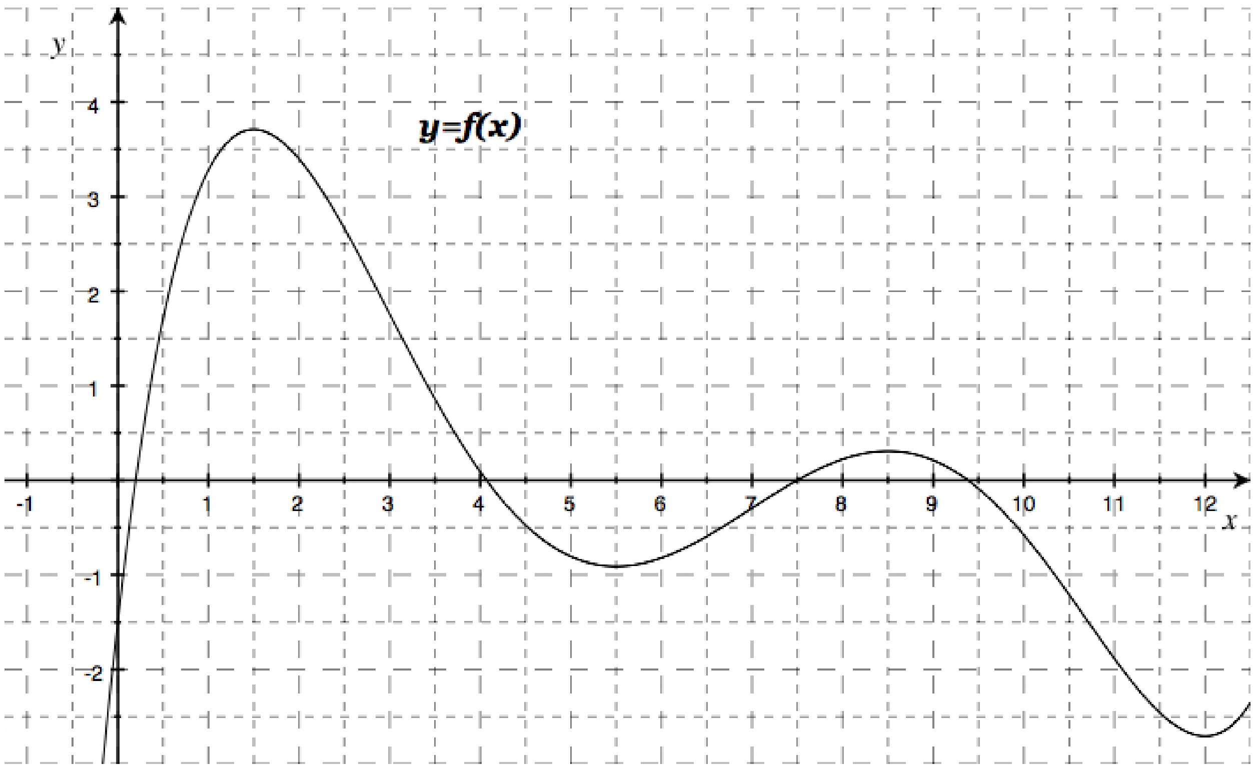
(c) Recall that $FC = TC(0)$, $TC(q) = VC(q) + FC$, and $AVC(q) = \dfrac{VC(q)}{q}$. Find all critical numbers of $AVC(q)$.

ANSWER: (list all) $q =$__________________thousand Gizmos

(d) Let $P(q)$ denote the profit (in thousands of dollars) at $q$ thousand Gizmos. The critical numbers of $P(q)$ are $q = 1.16$ and $q = 25.84$ thousand Gizmos. Determine whether each critical number gives a local minimum of $P(q)$, a local maximum of $P(q)$, or neither.

ANSWER: $q = 1.16$ gives a (circle one) local min local max neither
$q = 25.84$ gives a (circle one) local min local max neither

3. (18 points) Below is the graph of a function $y = f(x)$.




Define the function $A(m)$ by $A(m) = \int_0^m f(x)\,dx$.

NOTE: You do **not** need to show any work for the problems **on this page.**

(a) Name all values of $m$ at which $A(m)$ has a local minimum.

ANSWER: $m =$ ____________________

(b) Give the one-minute interval over which $A(m)$ increases the most.

ANSWER: from __________ to __________

(c) True or False?

**circle one**

**T** **F** $A(2.51) > A(2.50)$

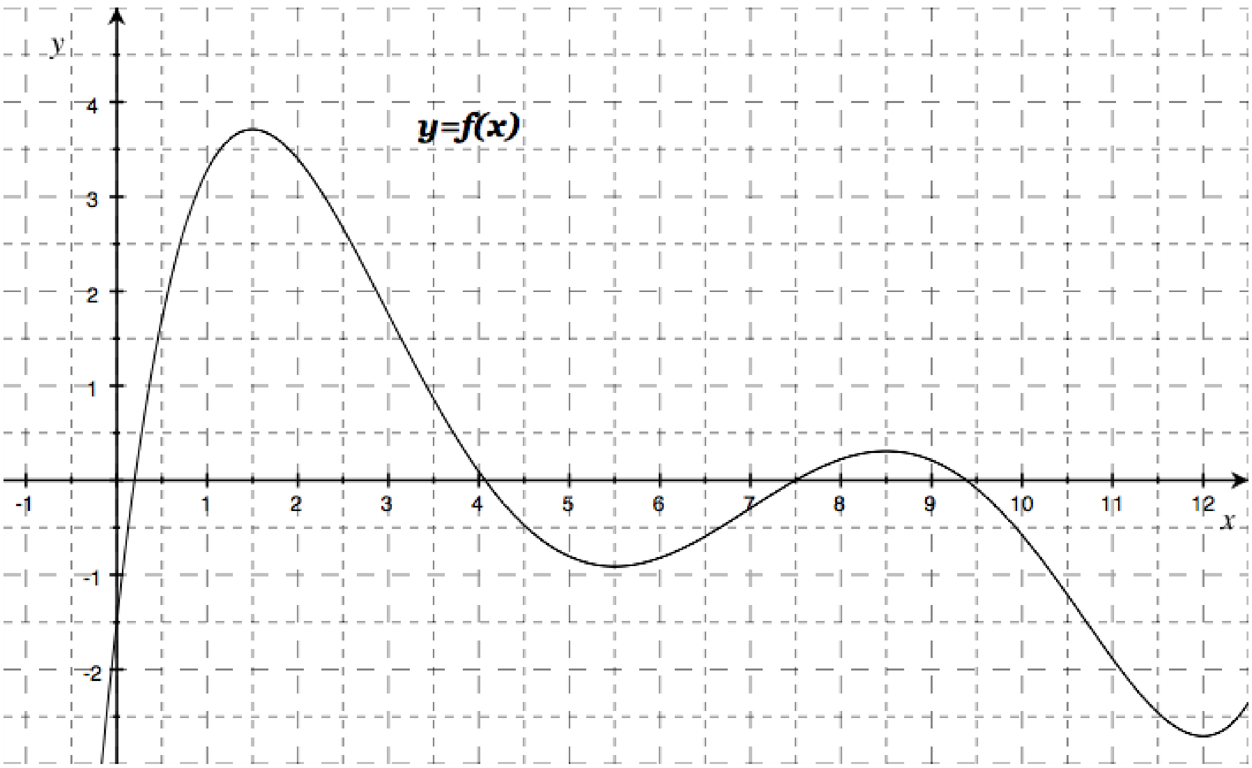
**T** **F** $f(2.51) > f(2.50)$

**T** **F** $A(10.01) > A(10.00)$

**T** **F** $f'(1.00) > f'(1.01)$

(THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.)

Here is the graph of $y = f(x)$ again.




And, again, $A(m) = \int_0^m f(x)\,dx$.

<u>NOTE:</u> The problems on this page **require some justification**: clearly mark points and lines on the graph, shade areas, show calculations of slopes and areas, etc.

(e) Compute $A(1)$.

ANSWER: $A(1) =$ ____________________

(f) Compute $A'(12)$.

ANSWER: $A'(12) =$ ____________________

(g) Compute $A''(5)$.

ANSWER: $A''(5) =$ ____________________

(h) Name a value of $x$ at which $f(x) = f(7)$.

ANSWER: $x =$ ____________________

(i) Compute $A(4) - A(2)$.

ANSWER: $A(4) - A(2) =$ ____________________

4. (13 pts) Your Total Cost (in hundreds of dollars) and Demand Curve (in dollars) *vs.* the quantity $q$ in hundreds of Items sold is given by the function:

$$TC(q) = \frac{q^3}{12} - \frac{q^2}{2} + \frac{3}{4}q + 10 \quad \text{and} \quad p = h(q) = 24 - 8\sqrt{q}.$$

(a) (7 pts) Write the formula for **Total Revenue**, $TR$, and give the **prices** that correspond to the global maximum and global minimum value of **Total Revenue** over the interval $q = 2$ to $q = 6$ hundred Items.

ANSWER: **PRICE** for the global minimum value = ________________ dollars

**PRICE** for the global maximum value = ________________ dollars

(b) (6 pts) Find **all** critical numbers of **Total Cost**, $TC$. Then use the second derivative test to determine whether $TC(q)$ reaches a local maximum, local minimum, or tell me if the test is inconclusive. Clearly put a box around your critical numbers and clearly label each as either local max, local min, or test inconclusive.

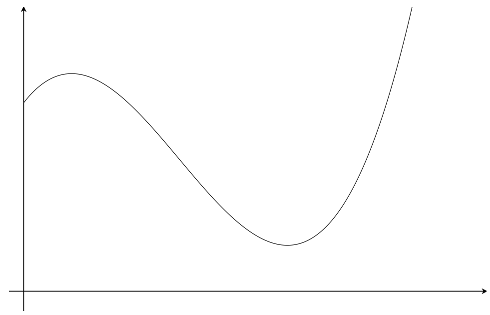
5. (15 points)

To the right is a sketch of the graph of the function

$$f(t) = 0.02t^3 - 0.39t^2 + 1.32t + 8.$$

The derivative of another function $g(t)$ is given by

$$g'(t) = 0.096t - 0.48.$$



(a) Find all values of $t$ at which the tangent line to $g(t)$ has slope 3.

ANSWER: $t =$____________________

(b) Find all values of $t$ at which the graph of $f(t)$ has a horizontal tangent line.

ANSWER: $t =$____________________

(c) Is $f(t)$ concave up or concave down at $t = 7$?

ANSWER: concave ____________________

(d) Suppose we are told that $f(11) = g(11)$. Find the formula for $g(t)$.

ANSWER: $g(t) =$________________________________________

(e) Which is smaller, A or B?

A. The lowest value of $g(t)$ on the interval from $t = 0$ to $t = 12$; OR

B. The lowest value of $f(t)$ on the interval from $t = 0$ to $t = 12$.

You must, as always, show some work to justify your answer.

ANSWER: ______________is smaller

6. (19 points) Let $f(x)$ be the function

$$f(x) = 5x^4 - 21x^3 + 24x^2 + 9x.$$

Define two new functions:

$$D(x) = \frac{f(x)}{x} = \text{the slope of a diagonal to } f(x)$$

and

$$T(x) = \text{the slope of a tangent to } f(x).$$

(a) Write out formulas for:

- $D(x) =$ ______________________________
- $D'(x) =$ ______________________________
- $D''(x) =$ ______________________________
- $T(x) =$ ______________________________
- $T'(x) =$ ______________________________
- $T''(x) =$ ______________________________

(b) Find all values of $x$ at which the function $D(x)$ has a horizontal tangent line.

ANSWER: $x =$ ________________ (list all)

(c) Apply the Second Derivative Test to the values you found in part (b) and state whether each critical number gives a local maximum or local minimum of $D(x)$.

(d) Find the largest and smallest values of $T''(x)$ on the interval from $x = 2$ to $x = 5$.

ANSWER: largest = ________________, smallest = ________________

7. (18 points) You sell Items. The total revenue and total cost (in **thousands of dollars**) for selling $q$ **thousand Items** are

$$TR(q) = \frac{2}{3}(q+25)^{3/2} - \frac{250}{3} \qquad TC(q) = \frac{1}{12}q^2 + 3q + 10.$$

(a) Find the positive quantity at which marginal revenue is equal to marginal cost.

ANSWER: $q =$ ________________ thousand Items

(b) Let $P(q)$ denote the profit for selling $q$ thousand Items. Find formulas for $P(q)$, $P'(q)$ and $P''(q)$.

ANSWER: $P(q) =$ ________________________

$P'(q) =$ ________________________

$P''(q) =$ ________________________

(c) Use the Second Derivative Test to determine whether the quantity you found in part (a) gives a local maximum or local minimum of $P(q)$.

(d) Recall that average cost is given by $AC(q) = \frac{TC(q)}{q}$. Determine whether average cost is concave up or concave down at $q = 50$. Show all your work.

ANSWER: (circle one) concave up concave down

8. (18 points) You sell Items. Your marginal revenue and marginal cost (both in dollars per Item) are given by the formulas

$$MR(q) = 200 - 5.46q \text{ and } MC(q) = 3q^2 - 48q + 197,$$

where $q$ is in **thousands of Items.**

(a) Find the quantity at which $TR$ changes from increasing to decreasing.

ANSWER: $q =$____________________thousand Items

(b) Find the quantity that maximizes profit.

ANSWER: $q =$____________________thousand Items

(c) Write out the formulas for total revenue and variable cost.

ANSWER: $TR(q) =$________________________________________

ANSWER: $VC(q) =$________________________________________

(d) If you sell $q = 8$ thousand Items, then your profit is 575.94 thousand dollars. Find the value of your fixed costs.

ANSWER: $FC =$____________________thousand dollars

(e) Compute the area under the $MC$ graph from $q = 2$ to $q = 10$.

ANSWER: ____________________

(f) Explain in English what the number you found in part (e) represents in terms of Total Cost.

9. (12 points)

Suppose that in order to achieve monthly sales of $q$ thousand Items you have to sell your Items at a price

$$p(q) = q^2 - 25q + 150 \text{ (dollars per Item)}$$

a) Determine all quantities for which your demand curve $p(q)$ is decreasing and not negative.
Justify your answer.

Answer: from $q$ =_______ to $q$ =_________ thousand Items.

b) Find all the critical points for your Total Revenue function. Round your answers to 2 decimal digits.

Answer: TR has critical points at $q$ = ______________________________thousand Items

c) Use the Second Derivative Test to determine whether each of the critical points you found in part (b) is a local maximum or a local minimum for the total revenue. Show all work and circle your answers.

10. (16 points) Water flows in and out of two vats, vat $A$ and vat $B$. The **rate of flow** (in gallons per minute) for vat $A$ at time $t$ minutes is given by the formula:

$$a(t) = 6t^2 - 66t + 144,$$

while the **amount** (in gallons) in vat $B$ at time $t$ minutes is given by the formula:

$$B(t) = 2t^2 - 30t + 65.$$

At $t = 0$, vat $A$ **contains exactly 60 gallons more than** vat $B$.

(a) Find the longest interval on which the water level in vat $A$ is decreasing.

ANSWER: from $t =$___________________to $t =$___________________minutes

(b) Determine all times at which the water level in vat $A$ is reaching a local minimum value.

ANSWER: $t =$___________________

(c) Write out the formula for the amount $A(t)$ in vat $A$ at time $t$.

ANSWER: $A(t) =$______________________________________

(THIS PROBLEM CONTINUES ON THE NEXT PAGE.)

Here are those formulas again.

The **rate of flow** (in gallons per minute) for vat $A$ at time $t$ minutes is given by the formula:

$$a(t) = 6t^2 - 66t + 144,$$

while the **amount** (in gallons) in vat $B$ at time $t$ minutes is given by the formula:

$$B(t) = 2t^2 - 30t + 65.$$

(d) What is the highest rate at which water is flowing into vat $B$ on the interval from $t = 7$ to $t = 10$?

ANSWER: ____________________gallons per minute.

(e) How much water flows into vat $A$ from $t = 1$ to $t = 3$?

ANSWER: ____________________gallons

11. (15 points) Consider the function

$$f(x) = \frac{x^3}{3} - 15x^2 + 200x$$

a) (3 pts) Compute all critical numbers of $f(x)$.

ANSWER: $x =$ ____________________ (list all)

b) (5 pts) For each of the points you found in part (a), use the Second Derivative Test to determine whether it is a local minimum or a local maximum. Show all your work and circle your answers.

c) (3 pts) Is the graph of $f(x)$ increasing or decreasing at $x = 0$? Justify.

ANSWER: ____________________

BECAUSE: ______________________________________________

d) (4 pts) Determine the minimum value of $f(x)$ on the interval from $x = 1$ to $x = 15$. Show work.

ANSWER: The minimum value of $f(x)$ on the given interval is ____________________

12. (16 pts) A balloon moves up and down. Its rate-of-ascent (speed) at time $t$ hours is given by the function

$$\boldsymbol{a(t) = \frac{t^2}{2} - 10t + 35} \text{ (in feet/hour)}$$

Let $A(t)$ denote the altitude of this balloon at $t$ hours.

a) Compute the change in the altitude of this balloon from 1 to 2 hours, $A(2) - A(1)$.

ANSWER: $A(2) - A(1) =$ _______________ feet

b) Suppose $A(6) = 442$ feet. Compute the initial altitude of this balloon, $A(0)$.

ANSWER: $A(0) =$ _______________ feet

c) Find the candidates for the local minimum and the local maximum of the altitude A(t) of the balloon. For each, use the second derivative test to determine if they are a local minimum or a local maximum.

ANSWER: $A(t)$ has a local minimum at $t =$ _______________ hours

local maximum at $t =$ _______________ hours

d) Suppose another balloon, balloon B, has an initial altitude of 200 feet, and its rate of ascent is given by

$$b(t) = -3t + 24.$$

Give the formula in terms of $t$ for the altitude $B(t)$ of the balloon B after $t$ minutes.

ANSWER: $B(t) =$ _________________________

13. (13 points) The formulas for three functions are:

$$f(x) = 2x^2 - 10x + 12, \quad g(x) = \frac{4}{3}x^3 - 26x^2 + 88x + 400, \quad \text{and} \quad h(x) = 8\ln(x) - 2x + 5.$$

(a) (5 pts) Find the global maximum and global minimum values of $g(x)$ over the interval $x = 0$ to $x = 10$.

ANSWER: MIN VALUE = ________________

MAX VALUE = ________________

(b) (4 pts) Find the longest interval on which $f(x)$ is decreasing and $g(x)$ is decreasing.

ANSWER: $x =$ ________________ to $x =$ ________________

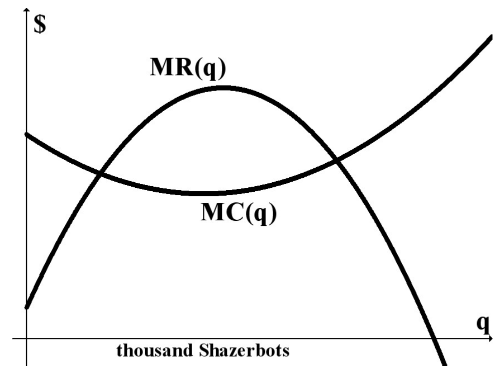
(c) (4 pts) Find the critical number(s) of $h(x)$. Then use the second derivative test to determine whether $h(x)$ reaches a local maximum, local minimum, or tell me if the test is inconclusive at the critical number(s) you found. Clearly show work and label answers.

14. (12 points)

You sell Shazerbots. The marginal revenue and marginal cost (each in dollars) in terms of $q$ thousand Shazerbots are given by the functions:

$$MR(q) = -3q^2 + 40q + 19, \text{ and}$$
$$MC(q) = q^2 - 12q + 124.$$

You are told that the Fixed Costs ($FC$) are \$15,000, so that $TC(0) = 15$. As always, $TR(0) = 0$.




**In each problem below, your final answers should have enough digits to be accurate to the nearest Shazerbot, or nearest cent.**

(a) (4 pts) Give the formulas for Total Revenue and Total Cost.

ANSWER: $TR(q) =$ ____________________

ANSWER: $TC(q) =$ ____________________

(b) (4 pts) Find the quantity at which Profit is maximum.

ANSWER: $q =$ ________________ thousand Shazerbots

(c) (4 pts) Recall that average cost is given by $AC(q) = \frac{TC(q)}{q}$. By making appropriate calculations with $AC(q)$ and its derivatives, determine if $AC(q)$ is concave up, concave down, or neither at $q = 2$.

ANSWER: (circle one) CONCAVE UP    CONCAVE DOWN    NEITHER

15. (13 points) Consider the function $f(t) = t^3 + 3t^2 - 9t + 700$.

a) (2 pts) Compute all values of $t$ at which the graph of $f(t)$ has a horizontal tangent line.

ANSWER: $t =$ ____________________ (list all)

b) (4 pts) For each of the points you found in part (a), use the second derivative test to determine whether it is a local minimum or local maximum. Show your work and circle your answers.

c) (i) (2 pts) Is the graph of $f(t)$ concave up or concave-down at $t = 7$? Justify.

ANSWER: It's concave-____________

BECAUSE: ______________________________________________________________

(ii) (2 pts) Is the point $t = 7$ a local minimum, local maximum, or neither for the function $f(t)$? Justify.

ANSWER (circle one): local minimum; local maximum; neither.

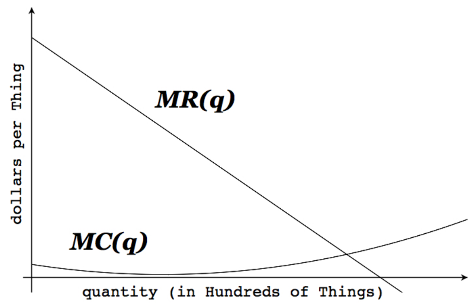
BECAUSE: ______________________________________________________________

d) (3 pts) Determine the maximum value of $f(t)$ on the interval from $t = -3$ to $t = 10$. Show work.

ANSWER: Max value of $f(t)$ on the given interval is ____________________.

16. (13 points)

You sell Things. The formulas for **Marginal Revenue** and **Marginal Cost** are graphed to the right and are given by
$MR(q) = 180 - 5q$
$MC(q) = 0.03q^2 - 0.9q + 8.75$,
where $q$ is measured in **Hundreds of Things** and $MR$ and $MC$ are in **dollars per Thing.** You also know your fixed costs: $FC = 4$ Hundred Dollars.




(a) Compute the change in **Total Revenue** that results from increasing quantity from 1 to 5 hundred Things.

ANSWER: ___________________Hundred Dollars

(b) Compute the **Profit** you earn if you produce 15 Hundred Things.

ANSWER: ___________________Hundred Dollars

(c) Compute the quantity that yields the largest profit.

ANSWER: ___________________Hundred Things

(d) Recall that **Average Cost** is $AC(q) = \dfrac{TC(q)}{q}$. Find the slope of the tangent line to $AC(q)$ at $q = 10$.

ANSWER: ___________________

17. (18 points) The demand curve for Trinkets has the formula

$$p = h(q) = 15 - 4\sqrt{q},$$

where $q$ is measured in Thousands of Trinkets and price $p$ is measured in Dollars per Trinket. You also know that variable cost to produce $q$ Thousand Trinkets is given by the formula:

$$VC(q) = 2q,$$

where $VC$ is measured in Thousands of Dollars.

(a) Write out formulas for total revenue, $TR(q)$, and its derivative, $TR'(q)$.

ANSWERS: $TR(q) =$ ____________________________

$TR'(q) =$ ____________________________

(b) Find all critical numbers of $TR(q)$.

ANSWER: $q =$ ________________

(c) Use the Second Derivative Test to determine whether your answer(s) to part (b) give a local maximum or local minimum of $TR(q)$.

(THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.)

(d) Find the quantity that maximizes profit. (HINT: Profit is maximized at a quantity at which marginal revenue is equal to marginal cost.)

ANSWER: $q =$__________________Thousand Trinkets

(e) If you sell 1 Thousand Trinkets, then your profit is 5.84 Thousand Dollars. What is the value of your fixed cost?

ANSWER: $FC =$__________________Thousand Dollars