V2

1. (14 pts) A company sells items. For all functions in this problem, x is in *thousands* of items. The selling price per item is $p = 57 - 8x$ dollars/item.
Total Cost, $TC(x)$, is a *linear* function with fixed cost of 14 thousand dollars and a cost of 32 thousand dollars to produce 3 thousands items. That is, $TC(0) = 14$ and $TC(3) = 32$.
Keep enough digits so final answers are accurate to the nearest item.

(a) (2 pts) Find the *linear* function for $TC(x)$.

$m = \frac{32-14}{3-0} = \frac{18}{3} = 6$ $\qquad TC(x) =$ $6x + 14$ thousand dollars

(b) (4 pts) Find the quantity when average cost, $AC(x)$, is 9 dollars per item.

$AC(x) = \frac{6x+14}{x} \stackrel{?}{=} 9$

$\Rightarrow \quad 6x + 14 = 9x$

$14 = 3x$

$x = \frac{14}{3} = 4.\overline{6}$

$x =$ 4.667 thousand items

(c) (4 pts) Find the larger quantity, x, where $TR(x)$ equals $VC(x)$.

$TR(x) = 57x - 8x^2 \stackrel{?}{=} 6x = VC(x)$

$0 = 8x^2 - 51x$ $\quad$ OR $\quad x = \frac{51 \pm \sqrt{51^2 - 0}}{2(8)}$ $\nearrow 0$ $\searrow \frac{102}{16} = 6.375$

$0 = x(8x - 51)$

$x = 0$ OR $8x - 51 = 0$

$x = \frac{51}{8} = 6.375$

$x =$ 6.375 thousand items

(d) (4 pts) Find the maximum profit. (Keep enough digits to be accurate to the nearest *dollar*)

$PROFIT = TR(x) - TC(x)$

$= (57x - 8x^2) - (6x + 14)$

$PROFIT = -8x^2 + 51x - 14$

$x = \frac{-51}{2(-8)} = 3.1875 \Rightarrow -8(3.1875)^2 + 51(3.1875) - 14$

$= 67.28125$

MAX PROFIT = 67.281 thousand dollars

V2

2. (14 points) For a different company. For all functions in this problem, x is in *hundred* of items. Total revenue is $TR(x) = 32x - 2x^2$ hundred dollars.
Fixed costs are \$1700 and average variable costs are $AVC(x) = 2.5x^2 - 19x + 42.75$ dollars/item.
Keep enough digits so final answers are accurate to the nearest item or nearest cent.

(a) (2 pts) Give the function for Total Cost, $TC(x)$.

$TC(x) =$ $2.5x^3 - 19x^2 + 42.75x + 17$ hundred dollars

(b) (4 pts) Find the shutdown price.

LOWEST y-VALUE OF AVC(x)

$x = -\frac{-19}{2(2.5)} = 3.8$

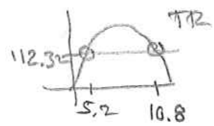
$y = AVC(3.8) = 2.5(3.8)^2 - 19(3.8) + 42.75$
$= 6.65$

16.65 dollars/item

(c) (4 pts) Find the largest interval of x-values where $TR(x)$ is greater than or equal to 112.32 hundred dollars.

$32x - 2x^2 \stackrel{?}{=} 112.32$

$0 = 2x^2 - 32x + 112.32$



$x = \frac{32 \pm \sqrt{32^2 - 4(2)(112.32)}}{2(2)} = \frac{32 \pm \sqrt{125.44}}{4} = \frac{32 \pm 11.2}{4}$ ↗ 5.20 ↘ 10.80

from $x =$ 5.20 to $x =$ 10.80 hundred items

(d) (4 pts) Find and completely simplify $MR(x) = \frac{TR(x + 0.01) - TR(x)}{0.01}$.

$\frac{[32(x+0.01) - 2(x+0.01)^2] - [32x - 2x^2]}{0.01}$

$= \frac{\cancel{32x} + 0.32 - 2(x^2 + 0.02x + 0.0001) - \cancel{32x} + 2x^2}{0.01}$

$= \frac{0.32 - 0.04x - 0.0002}{0.01}$

$= -4x + 32 - 0.02 = -4x + 31.98$

$MR(x) =$ $-4x + 31.98$ dollars/item

V2

3. (10 pts) Your company makes two kinds of cleaners: Miracle Cleaner and Speedex Cleaner.

Each gallon of Miracle Cleaner brings in \$2 dollars in profit and each gallon of Speedex Cleaner brings in \$2.50 dollars in profit.

You have enough supplies to make *at most* 960 gallons of Miracle Cleaner and *at most* 1150 gallons of Speedex Cleaner.
Also, your combined total gallons of both cleaners can be *at most* 1500 gallons.

Let x = 'gallons of Miracle Cleaner' and y = 'gallons of Speedex Cleaner'.

(a) Give the constraints and sketch/shade the feasible region.
You **must** label all x-intercepts, y-intercepts, and **intersection points** for full credit.

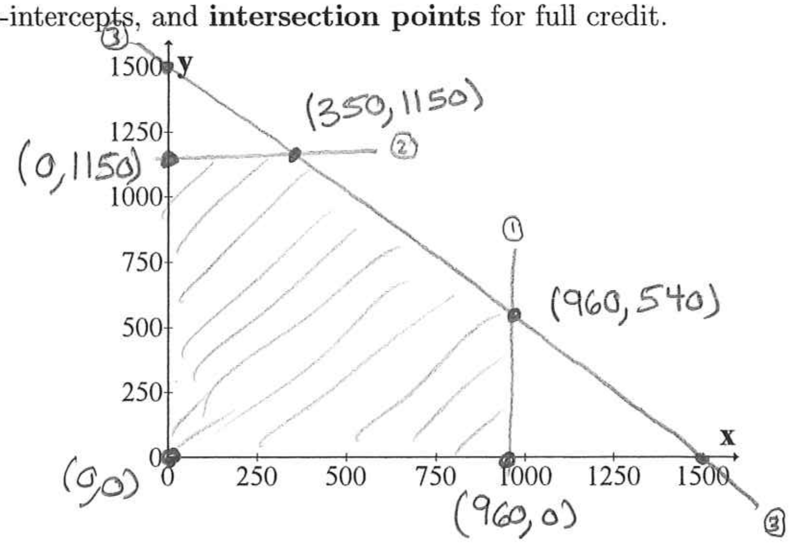
$x \leq 960$

$y \leq 1150$

$x + y \leq 1500$

① $x = 960$
③ $x + y = 1500$
$960 + y = 1500$
$y = 540$

② $y = 1150$
③ $x + y = 1500$
$x + 1150 = 1500$
$x = 350$



(b) How much of each type of mix should you produce to give maximum profit?
Also give the value of maximum profit? (Show your work)

PROFIT $= P(x,y) = 2x + 2.5y$

$P(0,0) = \$0$

$P(960,0) = 2 \cdot 960 = \$1920$

$P(0,1150) = 2.5 \cdot 1150 = \$2875$

$P(350,1150) = 2 \cdot 350 + 2.5 \cdot 1150 = \$3575$

$P(960,540) = 2 \cdot 960 + 2.5 \cdot 540 = \$3270$

$x =$ 350 gallons

$y =$ 1150 gallons

Max Profit = 3,575 dollars

V2

4. (10 pts)

(a) (6 pts) The demand function for a certain commodity is given by $p^2 + 12q = 1880$ and the supply function is given by $1000 - p^2 + 10q = 0$. Find the equilibrium quantity and equilibrium price (Round your final answers to two decimal places)

① $p^2 = 1880 - 12q$

② $1000 - p^2 + 10q = 0$

① \& ② $1000 - (1880 - 12q) + 10q \stackrel{?}{=} 0$

$$-880 + 22q = 0$$

$$q = \frac{880}{22} = 40$$

$p^2 = 1880 - 12(40)$

$p^2 = 1400$

$p = \pm\sqrt{1400} \approx 37.41657$

$q =$ 40

$p =$ 37.42

(b) (4 pts) Solve $5 + 4(2)^{3x} = 25$.
Give your final answer **as a decimal**, accurate to three digits after the decimal.

$4(2)^{3x} = 20$

$(2)^{3x} = 5$

$\ln(2^{3x}) = \ln(5)$

$3x \ln(2) = \ln(5)$

$$x = \frac{\ln(5)}{3\ln(2)} \approx 0.773976$$

$x \approx$ 0.774