V1

1. (14 pts) A company sells items. For all functions in this problem, x is in *thousands* of items. The selling price per item is $p = 46 - 8x$ dollars/item.
Total Cost, $TC(x)$, is a *linear* function with fixed cost of 16 thousand dollars and a cost of 31 thousand dollars to produce 3 thousands items. That is, $TC(0) = 16$ and $TC(3) = 31$.
Keep enough digits so final answers are accurate to the nearest item.

(a) (2 pts) Find the *linear* function for $TC(x)$.

$$m = \frac{31-16}{3-0} = \frac{15}{3} = 5$$

$TC(x) =$ $5x + 16$ thousand dollars

(b) (4 pts) Find the quantity when average cost, $AC(x)$, is 11 dollars per item.

$$AC(x) = \frac{5x+16}{x} \stackrel{?}{=} 11$$

$$5x + 16 = 11x$$

$$16 = 6x$$

$$x = \frac{16}{6} = 2.\overline{6} \approx 2.667$$

$x =$ 2.667 thousand items

(c) (4 pts) Find the larger quantity, x, where $TR(x)$ equals $VC(x)$.

$$TR(x) = 46x - 8x^2 \stackrel{?}{=} 5x = VC(x)$$

$$0 = 8x^2 - 41x \quad \text{OR} \quad x = \frac{41 \pm \sqrt{41^2 - 0}}{2(8)} = \frac{82}{16} = 5.125, \ 0$$

$$0 = x(8x - 41)$$

$$x = 0 \quad \text{OR} \quad 8x - 41 = 0$$

$$x = \frac{41}{8} = 5.125$$

$x =$ 5.125 thousand items

(d) (4 pts) Find the maximum profit. (Keep all digits to the end. Then keep enough digits in your final answer to be accurate to the nearest *dollar*)

$$\text{PROFIT} = TR(x) - TC(x)$$

$$= (46x - 8x^2) - (5x + 16)$$

$$\Rightarrow \text{PROFIT} = -8x^2 + 41x - 16$$

$$x = -\frac{41}{2(-8)} = 2.5625 \Rightarrow -8(2.5625)^2 + 41(2.5625) - 16$$

$$= 36.53125$$

MAX PROFIT = 36.531 thousand dollars

V1

2. (14 points) For a different company. For all functions in this problem, x is in *hundred* of items.
Total revenue is $TR(x) = 28x - 2x^2$ hundred dollars.
Fixed costs are \$2300 and average variable costs are $AVC(x) = 2.5x^2 - 11x + 31.25$ dollars/item.
Keep enough digits so final answers are accurate to the nearest item or nearest cent.

(a) (2 pts) Give the function for Total Cost, $TC(x)$.

$TC(x) =$ $2.5x^3 - 11x^2 + 31.25x + 23$ hundred dollars

(b) (4 pts) Find the shutdown price.

LOWEST y-VALUE OF AVC(x)

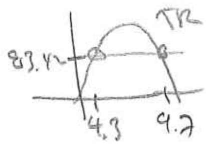
$$x = -\frac{-11}{2(2.5)} = 2.2$$

$$y = AVC(2.2) = 2.5(2.2)^2 - 11(2.2) + 31.25$$
$$= 19.15$$

19.15 dollars/item

(c) (4 pts) Find the largest interval of x-values where $TR(x)$ is greater than or equal to 83.42 hundred dollars.

$$28x - 2x^2 \overset{?}{=} 83.42$$
$$0 = 2x^2 - 28x + 83.42$$



$$x = \frac{28 \pm \sqrt{28^2 - 4(2)(83.42)}}{2(2)} = \frac{28 \pm \sqrt{116.64}}{4} = \frac{28 \pm 10.8}{4} = \begin{matrix} 4.3 \\ 9.7 \end{matrix}$$

from $x =$ 4.30 to $x =$ 9.70 hundred items

(d) (4 pts) Find and completely simplify $MR(x) = \dfrac{TR(x + 0.01) - TR(x)}{0.01}$.

$$\frac{\left[28(x+0.01) - 2(x+0.01)^2\right] - \left[28x - 2x^2\right]}{0.01}$$

$$= \frac{\cancel{28x} + 0.28 - 2(\cancel{x^2} + 0.02x + 0.0001) - \cancel{28x} + \cancel{2x^2}}{0.01}$$

$$= \frac{0.28 - 0.04x - 0.0002}{0.01} = -4x + 28 - 0.02 = -4x + 27.98$$

$MR(x) =$ $-4x + 27.98$ dollars/item

V1

3. (10 pts) Your company makes two kinds of cleaners: Miracle Cleaner and Speedex Cleaner.

Each gallon of Miracle Cleaner brings in \$3 dollars in profit and each gallon of Speedex Cleaner brings in \$2.50 dollars in profit.

You have enough supplies to make *at most* 960 gallons of Miracle Cleaner and *at most* 1150 gallons of Speedex Cleaner.
Also, your combined total gallons of both cleaners can be *at most* 1500 gallons.

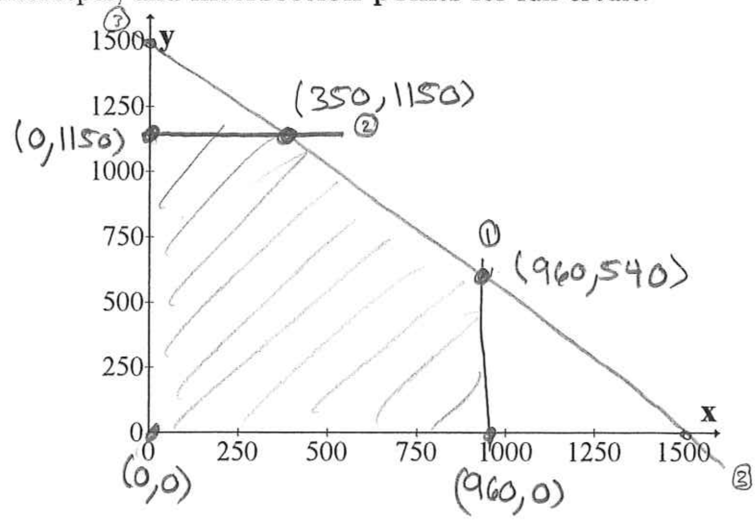
Let x = 'gallons of Miracle Cleaner' and y = 'gallons of Speedex Cleaner'.

(a) Give the constraints and sketch/shade the feasible region.
You **must** label all x-intercepts, y-intercepts, and **intersection points** for full credit.

$x \le 960$

$y \le 1150$

$x + y \le 1500$



① $x = 960$
③ $x+y = 1500$

$960 + y = 1500$
$y = 540$

② $y = 1150$
③ $x+y = 1500$

$x + 1150 = 1500$
$x = 350$

(b) How much of each type of mix should you produce to give maximum profit?
Also give the value of maximum profit? (Show your work)

PROFIT $= P(x,y) = 3x + 2.5y$

$P(0,0) = \$0$

$P(960,0) = 3 \cdot (960) = \$2880$

$P(0,1150) = 2.5 \cdot (1150) = \$2875$

$P(350,1150) = 3(350) + 2.5(1150) = \$3925$

$P(960,540) = 3 \cdot (960) + 2.5(540) = \$4230$

$x =$ 960 gallons

$y =$ 540 gallons

Max Profit = 4,230 dollars

V1

4. (10 pts)

(a) (6 pts) The demand function for a certain commodity is given by $p^2 + 12q = 1800$ and the supply function is given by $700 - p^2 + 10q = 0$. Find the equilibrium quantity and equilibrium price (Round your final answers to two decimal places)

① $p^2 = 1800 - 12q$

② $700 - p^2 + 10q = 0$

① & ② $700 - (1800 - 12q) + 10q \overset{?}{=} 0$

$\underbrace{700 - 1800}_{-880} + 12q + 10q = 0$

$22q = 1100$ -880

$q = \frac{1100}{22} = 50$

$p^2 = 1800 - 12(50) = 1200$

$\Rightarrow p = \pm\sqrt{1200}$

≈ 34.641016

$\boxed{q = 50 \quad p = 34.64}$

(b) (4 pts) Solve $5 + 4(2)^{3x} = 15$.
Give your final answer **as a decimal**, accurate to three digits after the decimal.

$4(2)^{3x} = 10$

$2^{3x} = \frac{10}{4} = 2.5$

$\ln(2^{3x}) = \ln(2.5)$

$3x \ln(2) = \ln(2.5)$

$x = \frac{\ln(2.5)}{3 \cdot \ln(2)} \approx 0.44064298$

$\boxed{x \approx 0.441}$