

Math Final Exam Quick Review

Exam 1 Material (Reading Graphs and Rates):

The main concept was rates (*i.e.* slopes) and how to graphically work with rates. There were two types of questions about rates:

1. **Compute a rate:** Draw the appropriate line, find two easy to read points, compute the slope.
2. **Given a Slope, find a time or interval:** Draw a reference line, then use it to answer the questions. You 'slide' your ruler parallel when you are trying to find a secant or tangent line that has the same slope as the reference line.

That really is all there is to it!

We also introduced business terms and learned some particular application situations. Special terms:

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|--------------------------|----------|---------|--------------|--------------------------|
| total amount = | distance | $TC(q)$ | $VC(q)$ | $TR(q)$ |
| slope of diagonal line = | $ATS(t)$ | $AC(q)$ | $AVC(q)$ | $AR(q)$ = price per item |
| slope of secant = | $AS(t)$ | $MC(q)$ | also $MC(q)$ | $MR(q)$ |

Important Applications:

- *Find quantities when the profit is zero.* This is the same as asking when is $TR = TC$. So you need to find the values of q at which TR and TC intersect.
- *Find the quantity at which profit is maximum (or any max gap question).* We discussed three methods:
 1. METHOD 1: Find the location of the largest gap when TR is above TC . (Hard to be precise with this method, but it works).
 2. METHOD 2: Find when $MR = MC$. (Good if you have a table of MR/MC values or if you have MR/MC graphs).
 3. METHOD 2': Match Slopes! That is, find when the slope of tangent to TR is the same as the slope of the tangent to TC . (This is just the graphical way of saying $MR = MC$, but it is the easiest and most precise method to use when given graphs of TR and TC).
- *Find the Breakeven Price (BEP).* Fix your ruler at the origin and rotate to the lowest diagonal line that touches the TC graph. Find the slope of this line! (Get two points and find the slope). Remember, this slope represents a transitional market price. If the market price is above the BEP, then it is *possible* to make a positive profit. If the market price is below the BEP, then it is NOT possible to make a positive profit.
- *Find the Shutdown Price (SDP).* Fix your ruler at the origin and rotate to the lowest diagonal line that touches the VC graph. Again, find the slope of this line! Recall: If the market price is above the SDP, then it is *possible* to recover some of your fixed costs (stay open). If the market price is below the SDP, then you are going to lose more money than your fixed costs no matter how many items you sell (shut down).

Exam 2 Material (Algebra, Functions, and Rates):

We learned some basic functions and algebra skills that are common in business:

- Lines: Get two points, compute the slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$ and use $y = m(x - x_1) + y_1$ to get $y = mx + b$.
- Parabolas: Given $y = ax^2 + bx + c$ we learned how to determine if the parabola opens upward ($a > 0$) or downward ($a < 0$) and we learned how to get the vertex (which is at $x = -\frac{b}{2a}$). We also learned about a tool to solve equations that involve quadratics (the quadratic formula).
- Exponentials: Given $y = Pe^{rt}$, we learned what this function looked like and we learned how to solve equations involving this function (how to use the natural logarithm).
- Solving a System of Equations: Solve for one variable in one equation and substitute into the other equation.

Important Applications:

- **Supply-Demand Problems:** Market equilibrium occurs where supply and demand intersect (so you need to solve a system of equations). If price is above market equilibrium, then there is a surplus. And if price is below market equilibrium, then there is a shortage. Know how to solve the system to find market equilibrium and understand the basics of this scenario.
- **Know Function Definitions and How to Use Functional Notation:** You should know how to go from price to TR , how to go from AC to TC , how to go from AVC to VC , how to go from TR/TC to MR/MC , how to go from Distance to ATS , etc.... In other words you need to know the function definitions and how to use them.
- **Remember the Standard Applications We Have Been Discussing All Term:** Maximum profit, break even quantity, break even price (BEP), shut down price (SDP), etc...
- **Know How to Do Linear Programming:** You need to be able to read information to get constraints and an objective. You need to be able to graph inequalities. You need to be able to **solve a system of equations** to get corners. And you need know how to finish the problem (evaluate the objective at each of the corners).

New Material (Interest Bearing Accounts):

Lump Sum Accounts: You deposit money today and make no more payments, you let interest grow, then you want to know the future value.

| | |
|------------------------------|--|
| Simple Interest | $F = P(1 + rt)$ |
| Compounding m times a year | $F = P \left(1 + \frac{r}{m}\right)^{mt} = P(1 + i)^n$ |
| Continuous Compounding | $F = Pe^{rt}$ |

Annuity Accounts: Regular payments (deposits/withdrawals) with an interest bearing account.

Two types of Annuities:

- Ordinary Annuities: Payments made at the END of each compounding period. Note: Loans are ordinary annuities.
- Annuities Due: payments made at the BEGINNING of each compounding period.

Two types of questions:

- Future Value Questions: start with zero dollars in the account, regular **deposits**, find the future value. The value in the account is growing!
- Present Value Questions: start with a lot of money (call this P), make regular **withdrawals**, end with zero in the account. The value in the account is shrinking! Note: Loan questions are present value questions (you are given a chunk of money and you are paying down the balance with interest to get to zero owed).

In these formulas, $i = \frac{r}{m}$ and $n = mt$.

| | Ordinary (END) | Due (BEGINNING) |
|--------------------|---|---|
| Future Value = ?? | $F = R \left[\frac{(1+i)^n - 1}{i} \right]$ | $F = R \left[\frac{(1+i)^n - 1}{i} \right] (1+i)$ |
| Present Value = ?? | $P = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$ | $P = R \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)$ |