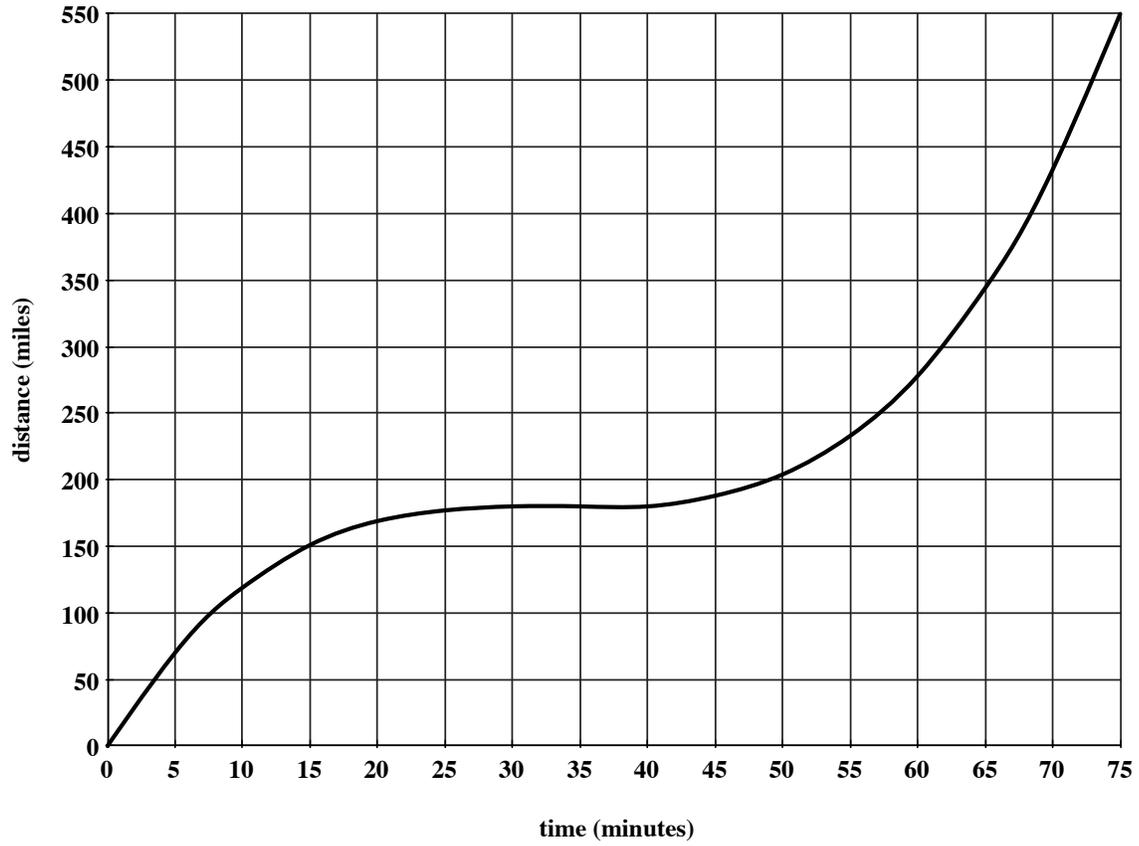


**GRAPHS AND EXAMPLES  
FOR LECTURE  
MATH 111**

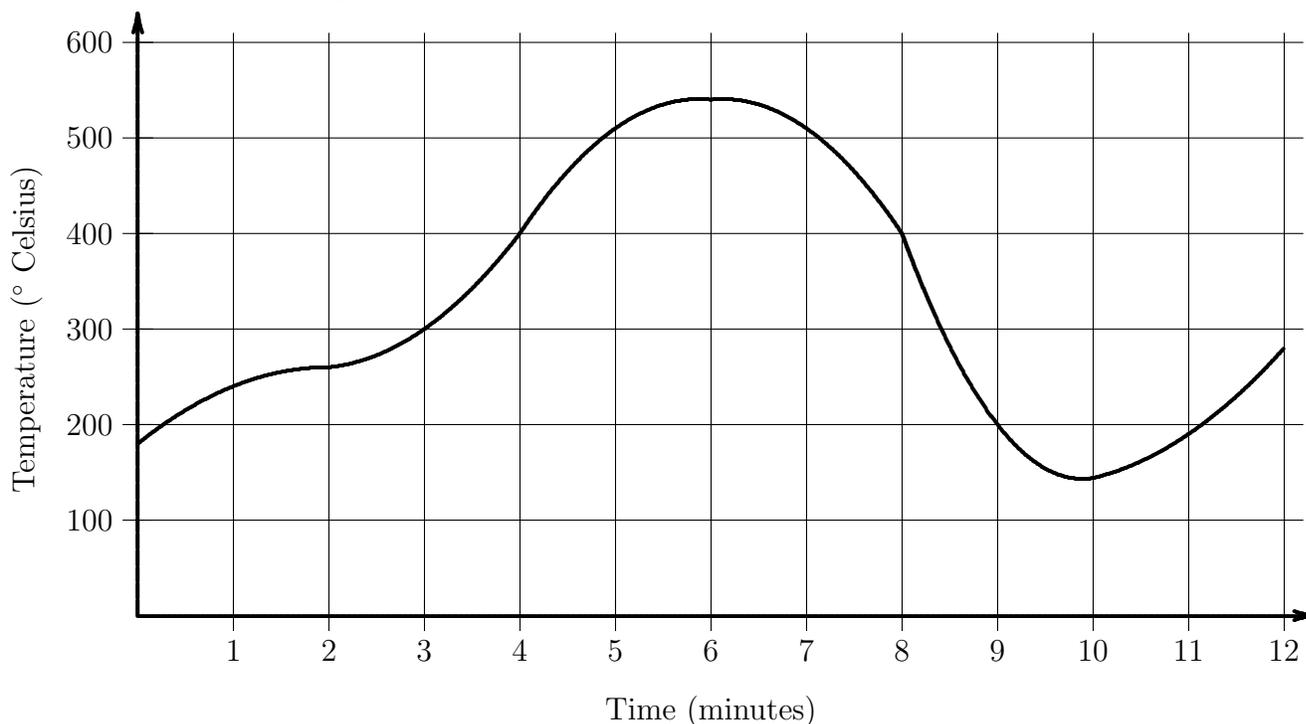
Lecture Materials for Supplement Sections 1 and 2

**Situation:** This is the graph of distance traveled for a car driving on a long straight road.



Lecture Materials for Supplement Sections 3 and 4

**Situation:** This is the graph of temperature vs. time for a chemical reaction.

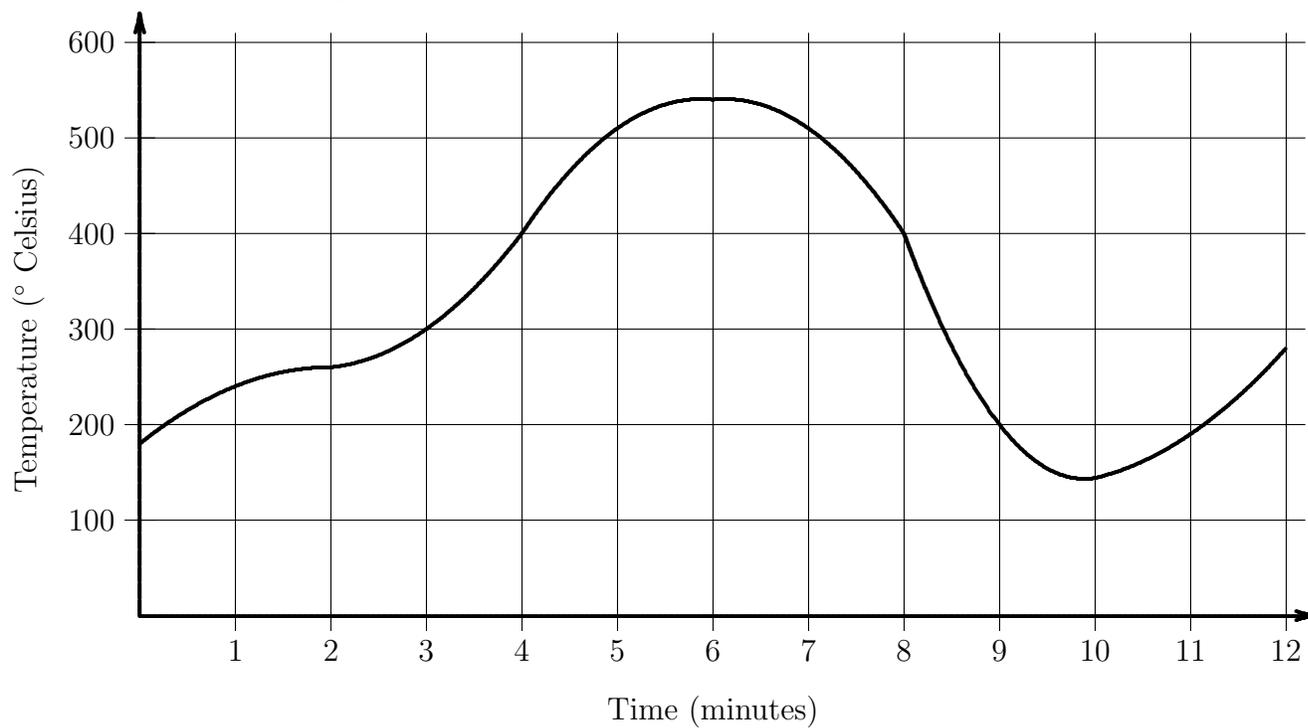


**Questions:**

1. What is the overall rate of change in temperature after 9 minutes?
2. How fast does the temperature rise, on average, during the 3-minute interval beginning at  $t = 4$  minutes?
3. Find a time at which the overall rate of change of temperature is 25 degrees per minute.
4. Find a two-minute interval during which the incremental rate of change of temperature is 10 degrees per minute. (That is, find a two-minute interval over which the temperature rises, on average, 10 degrees per minute.)
5. Find a three-minute interval during which the temperature rises by 150 degrees.

Lecture Materials for Supplement Section 5

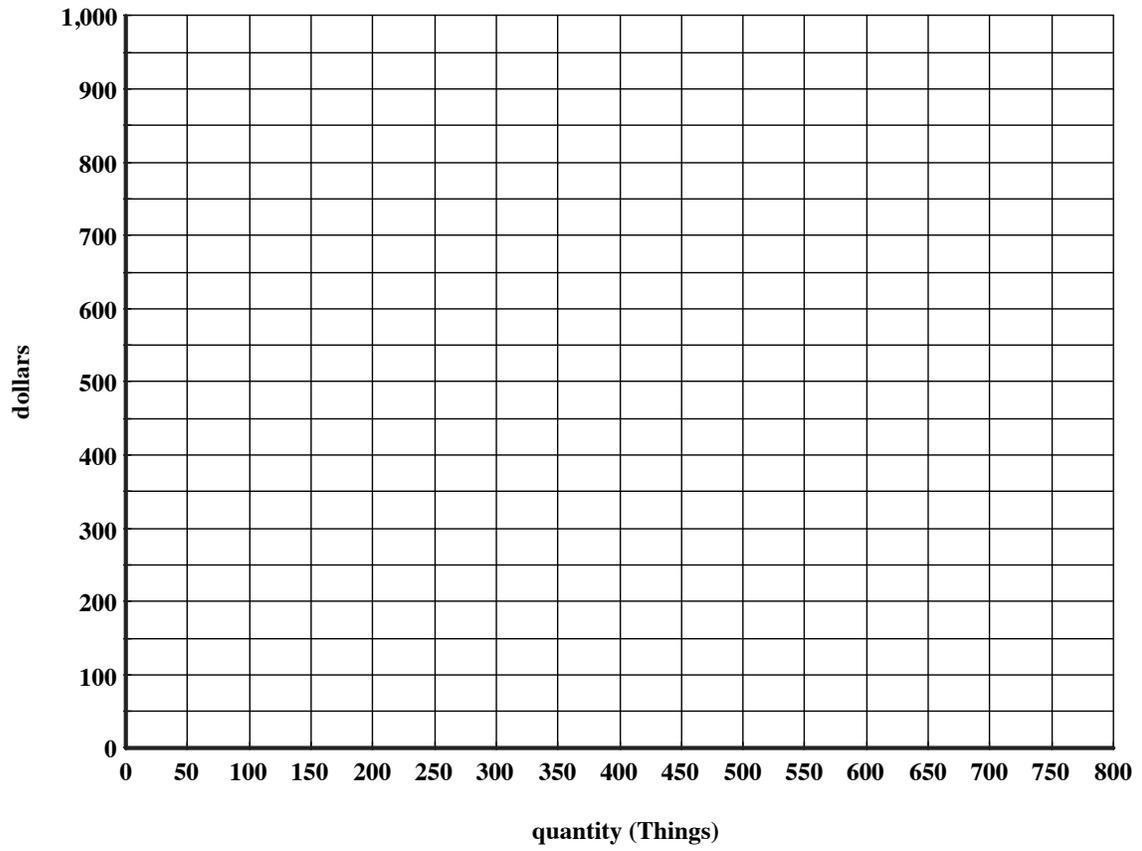
**Situation:** This is the graph of temperature vs. time for a chemical reaction.



	English	Graph	Functional Notation
1	At time $t = 4$ , the temperature is $400^\circ$ .	At $t = 4$ , the height of graph is 400.	$P(4) = 400$
2			$P(b) - P(a)$
3	the incremental rate of change in temperature from time $a$ to time $b$		
4		the “ $y$ ”-intercept of the temperature graph	
5			$\frac{P(b)}{b}$
6			$\frac{P(b)-P(0)}{b-0}$
7			$P(10) > P(9)$ * * false
8	Between 4 and 6 minutes, the temperature rises by $140^\circ$ .		
9		The slope of the secant from $t = 0$ to $t = 4$ is 57.	
10	When is the temperature $350^\circ$ ?		
11		The graph has height 200 for three different values of $t$ .	
12			Find $t$ so that $P(t) - P(2) > 100$ .
13	Find two times, 2 minutes apart, when the temperature is the same.		
14		slope of the secant line from 2 to $h$	

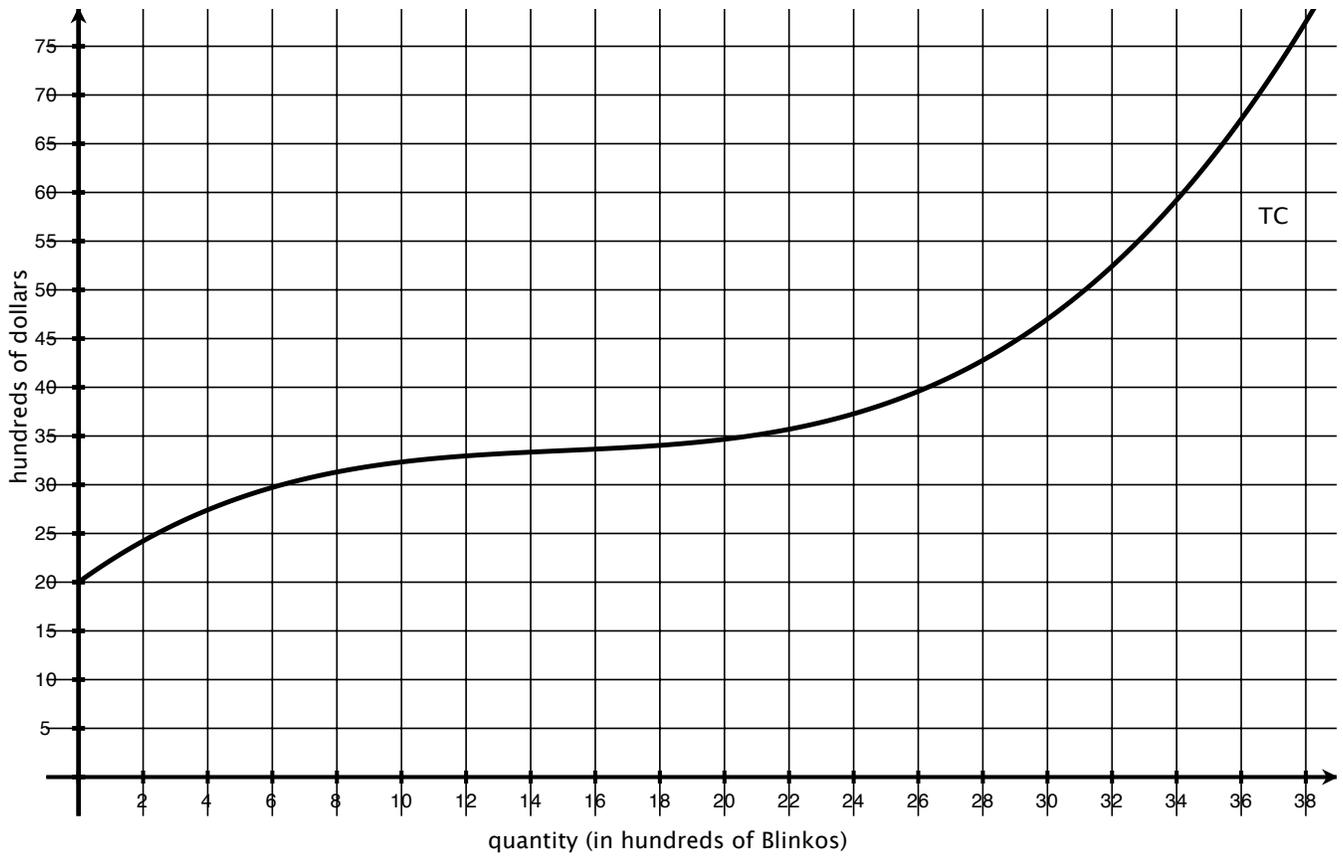
	English	Graph	Functional Notation
15	How many minutes after $t = 4$ does the temperature become $250^\circ$ ?		
16			If $\Delta t = 3$ , for what $t$ is $\Delta P$ highest?
17		the change in height of the graph between $t$ and $t + 2$	
18			Solve $P(3+r) - P(3) = 100$ for $r$
19		slope of secant line from $t$ to $t + 3$ .	
20	the average rate of change of temperature for $h$ minutes beginning at $t = 3$		
21		slope of the secant from $t = p$ to $t = q$ .	
22			Is $P(2) + P(3) = P(5)$ ?
23		For which $t$ is graph twice as high as it is when $t = 10$ ?	
24	Find a span over which temp rises by $50^\circ/\text{min}$ on average.		
25			$\frac{P(5)-P(2)}{5-2} > \frac{P(8)-P(2)}{8-2}$

Lecture Materials for Supplement Section 6



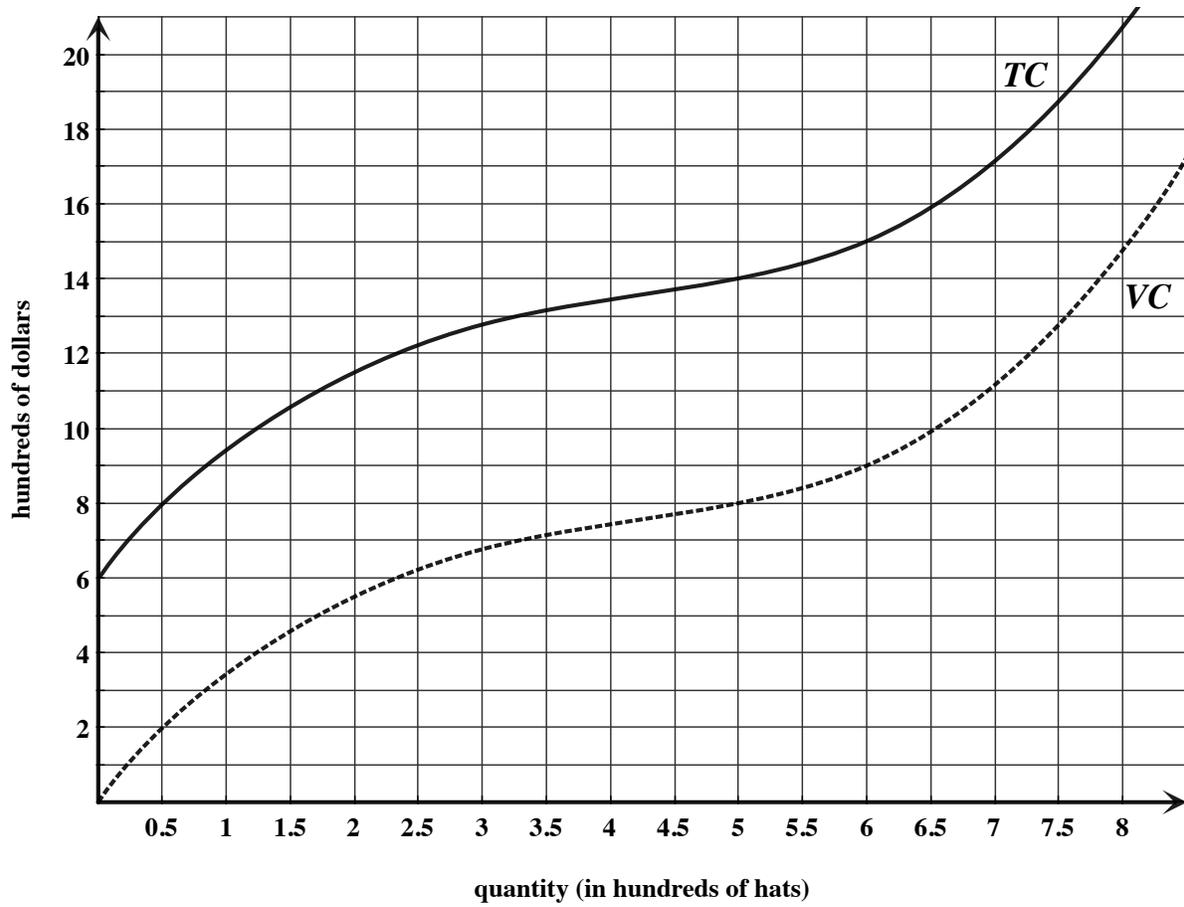
Lecture Materials for Supplement Section 7

**Situation:** This is the graph of total cost ( $TC$ ) for selling Blinkos. Note the units on the axes: quantity is measured in **hundreds of Blinkos** and  $TC$  is measured in **hundreds of dollars**.



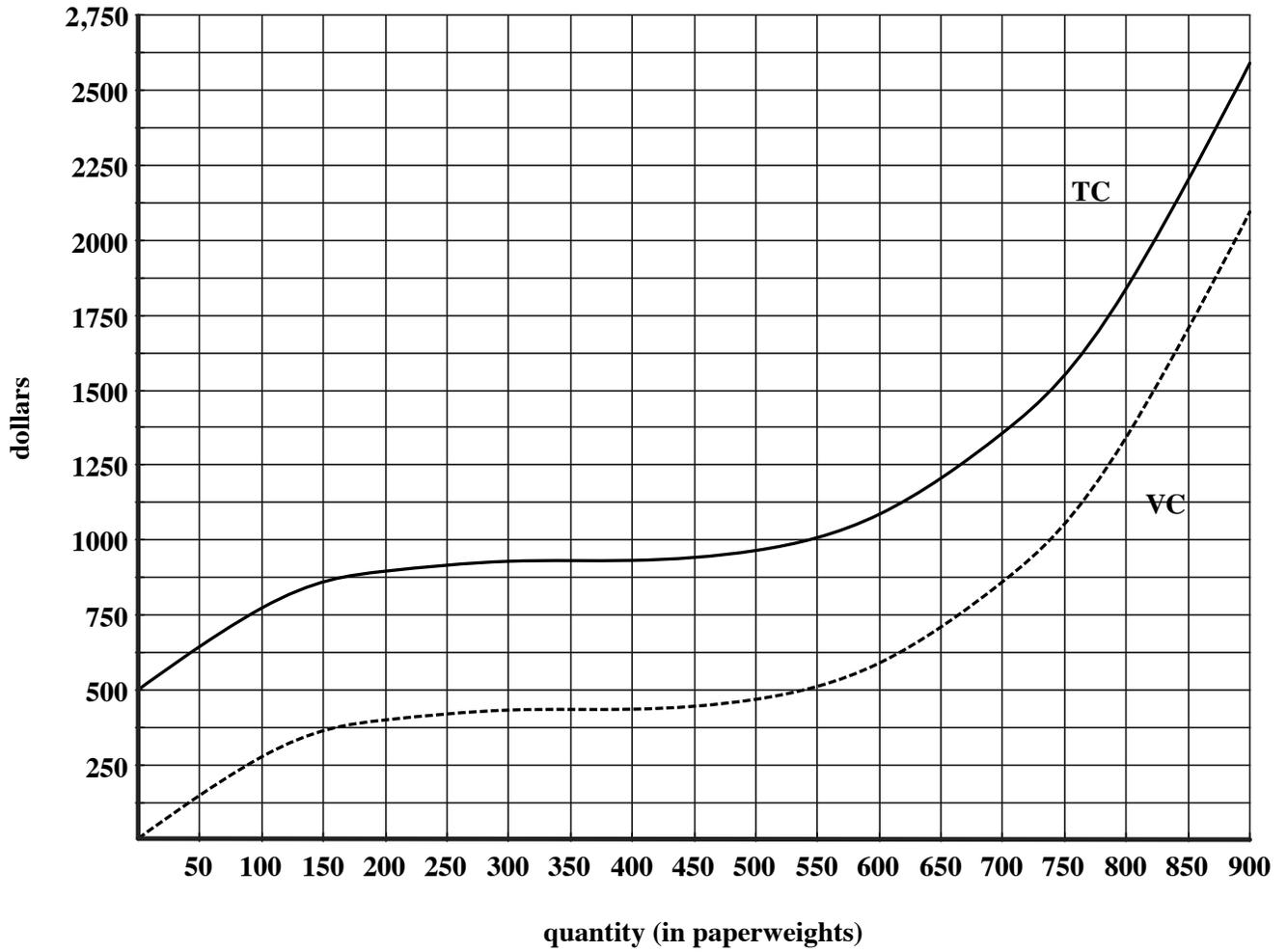
Lecture Materials for Supplement Section 8

**Situation:** The graphs below show total cost ( $TC$ ) and variable cost ( $VC$ ) for selling hats.

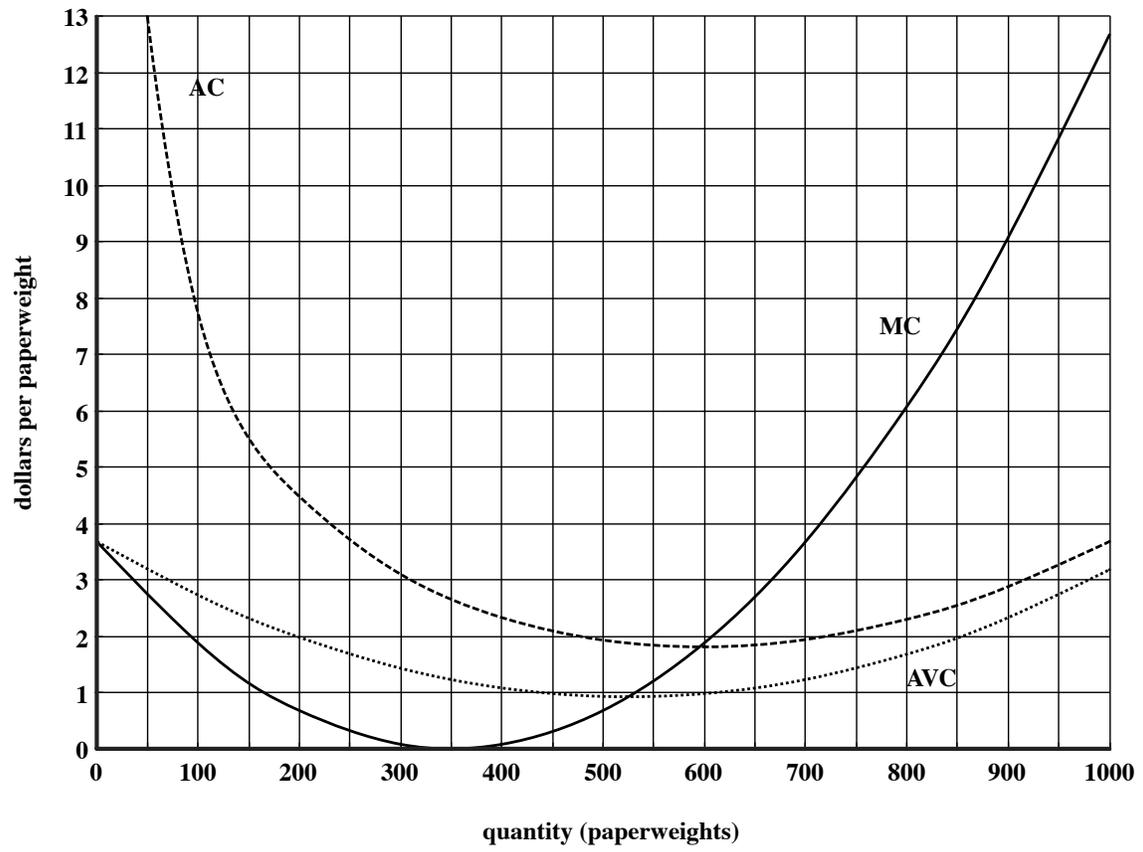


## Lecture Materials for Supplement Section 9

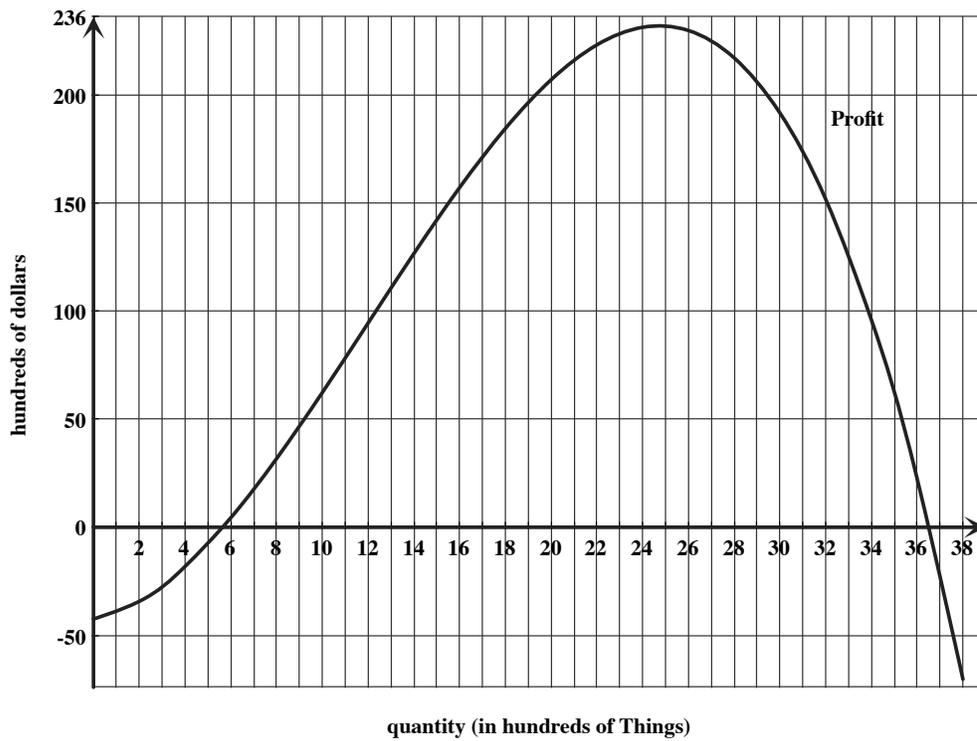
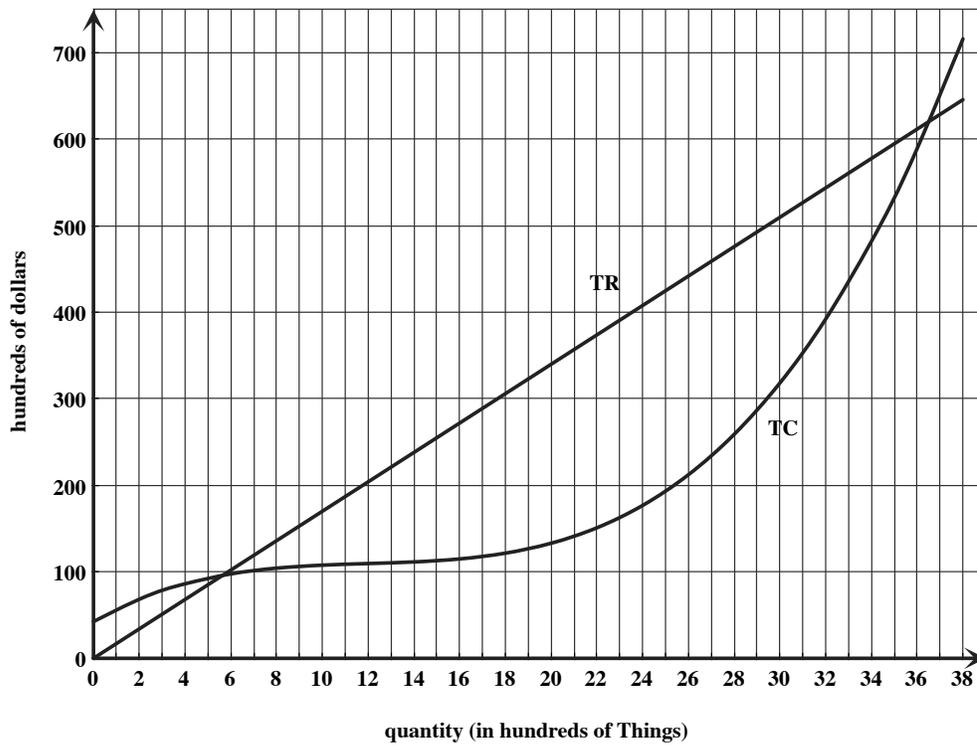
The following are the graphs of Total Cost and Variable Cost for selling paperweights.

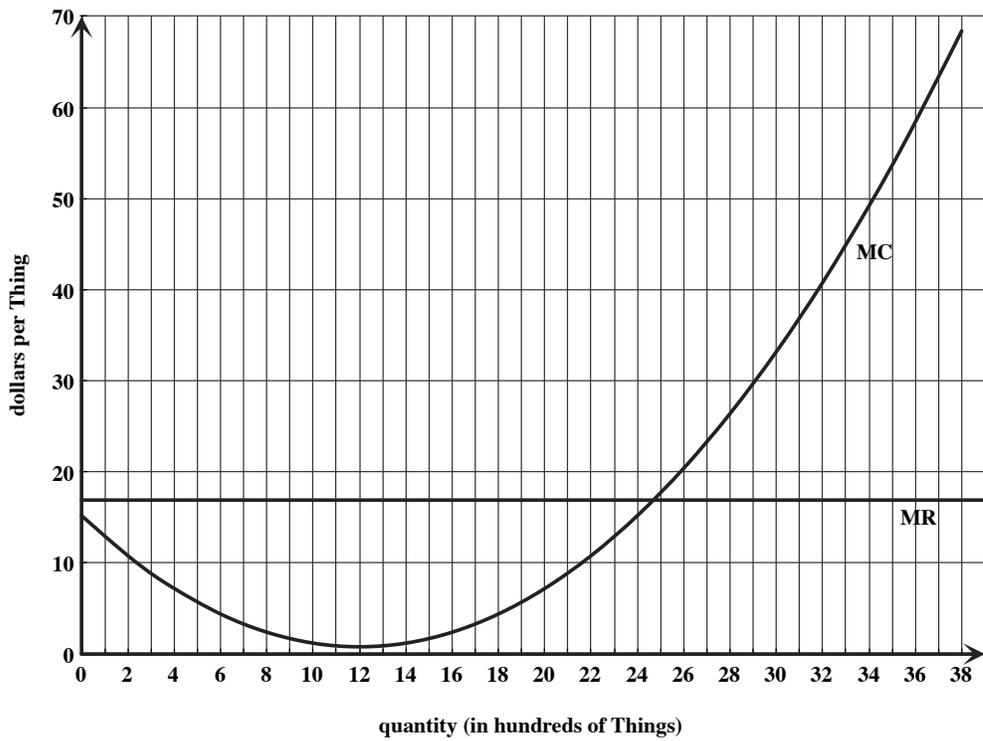
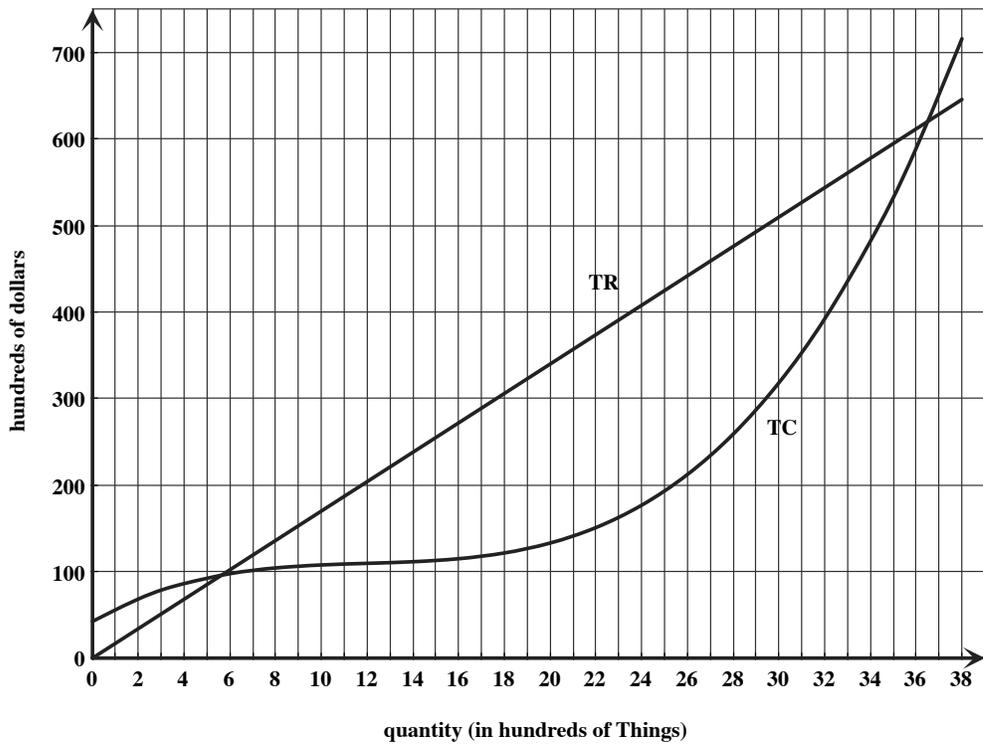


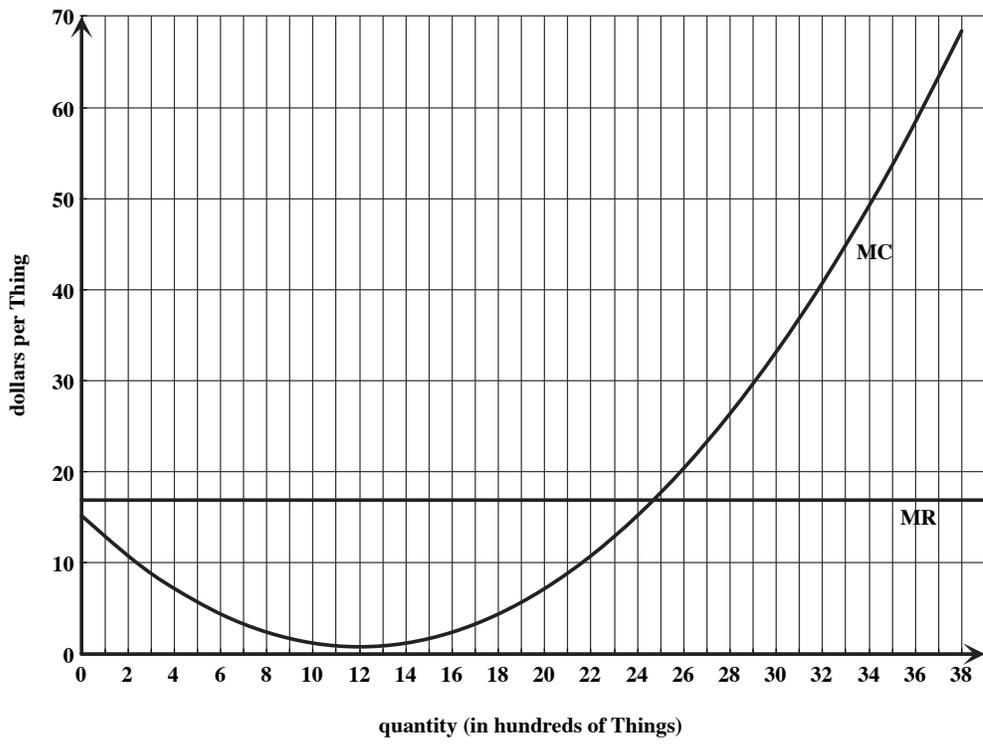
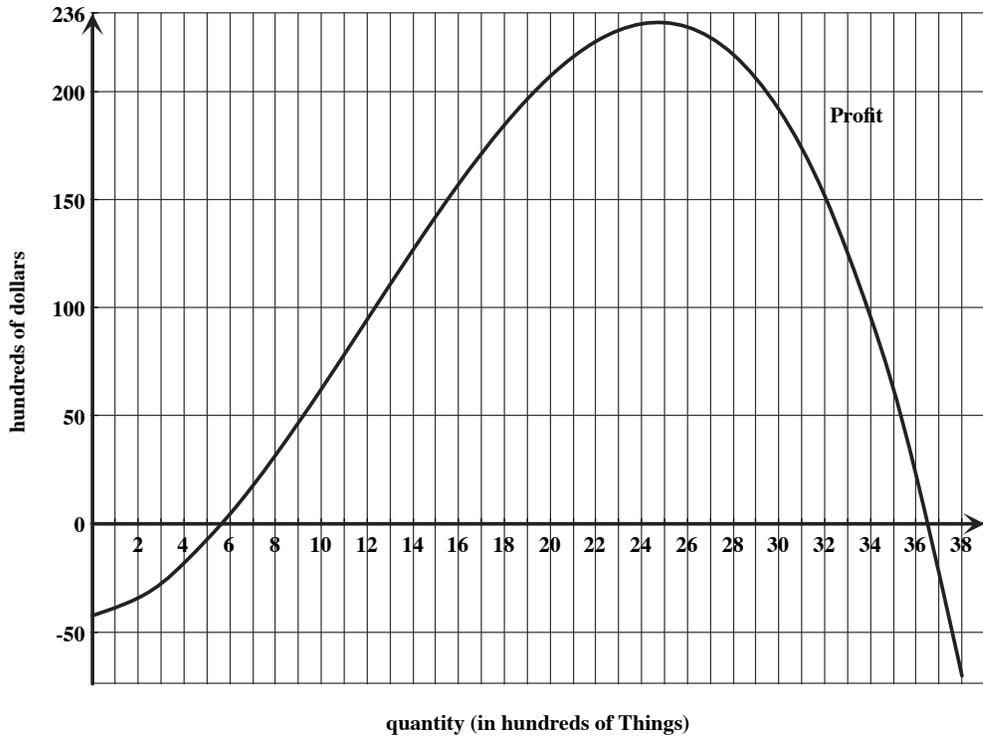
$q$	100	200	300	400	500	600	700	800	900	1000
$MC$	1.88		0.08	0.08	0.68	1.88	3.68	6.08	9.08	12.68
$AC$	7.73	4.48		2.33	1.93	1.81	1.94	2.30	2.88	3.68
$AVC$	2.73	1.98	1.43	1.08	0.93	0.98		1.68	2.33	3.18



Lecture Materials for Summary of Revenue, Cost, and Profit







Examples for Lecture — Section 1.3

1. You have \$25,000 to invest in two different funds. One investment fund has an annual yield of 8% and the other 11%. How much must you invest in each to earn \$2500 in one year?
2. To earn a grade of 3.0 or above in a course, a student must obtain at least an 85% average on two midterms and a final exam. The final exam is worth twice as much as the midterms. The student earns a 78 and a 93 on the midterms. How high must the student's final exam grade be to ensure a 3.0 in the course?
3. You sell Things. Your total cost, in hundreds of dollars, for producing  $x$  hundred Things is given by the formula

$$C(x) = 40 + 0.5x.$$

How much do you pay in fixed costs?

What is your total cost if you produce 700 Things?

How many Things are you producing if your total costs are \$12,000?

4. You sell Objects. Each object sells for \$5. Your total cost function is linear. When you produce 1 Object, total cost is \$53. When you produce 10 Objects, your total cost is \$80.
  - (a) Find formulas for:
    - Total revenue for selling  $x$  Objects
    - Total cost for producing  $x$  Objects
    - Profit for producing and selling  $x$  Objects
  - (b) What are:
    - fixed costs?
    - marginal revenue?
    - marginal cost?
    - marginal profit?
  - (c) For what levels of production (values of  $x$ ) do you make a profit?

Lecture Examples — HR §4.2: Linear Programming

1. Your company makes two fruit juices, Apple-Cranberry and Cranberry-Apple. Apple-Cranberry is 60% apple juice and 40% cranberry juice. Cranberry-Apple is 30% apple and 70% cranberry. You make \$0.40 profit on each gallon of Apple-Cranberry and \$0.50 profit on each gallon of Cranberry-Apple. Your daily supply is limited to 12,000 gallons of pure apple juice and 11,000 gallons of pure cranberry juice.

Let  $x$  be the amount of Apple-Cranberry juice, in gallons, you produce daily and  $y$  be the amount of Cranberry-Apple juice, in gallons, you produce daily. Determine how much of each you should produce daily to maximize your profit.

2. Gina inherits a large sum of money and a bunch of pet cages from an animal-loving aunt. She decides to rescue some unwanted pets from a shelter. She has 20 cages that can each house either a bunny or a ferret. She does some research and finds that, on average it costs \$0.60 a day to feed one ferret and \$0.80 a day to feed one bunny. Gina can budget no more than \$14.40 a day for pet food. But cuddliness is an issue for Gina. She figures that bunnies are twice as cuddly as ferrets. That is, ferrets are each worth one cuddle-unit, while bunnies are each worth two.

Determine how many of each pet Gina should adopt to maximize cuddliness while staying within her budget and without buying more cages.

3. (Harshbarger/Reynolds §4.2 #40) Two foods contain only proteins, carbohydrates, and fats. Food A costs \$1 per pound and contains 30% protein and 50% carbohydrates. Food B costs \$1.50 per pound and contains 20% protein and 75% carbohydrates. What combination of these two foods provides at least 1 pound of protein, 2.5 pounds of carbohydrates, and 0.25 pounds of fat at the lowest cost?

4. Your company makes two household cleaners: Miracle Bathtub Cleaner and Speedex Floor Cleaner. Your daily production of both cleaners combined is limited to 2,000 gallons. Your daily sales of Miracle Bathtub Cleaner never exceed 1,200 gallons, and your daily sales of Speedex Floor Cleaner never exceed 1,400 gallons. Finally, you make \$1.00 profit on each gallon of Miracle Bathtub Cleaner that you sell and \$2.00 on each gallon of Speedex Floor Cleaner that you sell.

Determine the amount of each cleaner you should produce in order to maximize profit.

## Examples for Lecture: HR §6.2–6.4

### Annual Percentage Yield

You are choosing a savings account and have the following options:

- (a) a nominal rate of 4%, compounded semi-annually
- (b) a nominal rate of 3.97%, compounded monthly
- (c) a nominal rate of 3.955%, compounded continuously

Which should you choose?

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### Annuities

An **annuity** is a financial plan characterized by regular payments. Annuities will involve a payment of  $\$R$  made  $m$  times a year to or from an account earning  $r \times 100\%$  interest, compounded  $m$  times a year, for  $t$  years. (The variables  $r$ ,  $m$ , and  $t$  are the same as in the compound interest formulas.) From these, we obtain two new variables: the compounding rate,  $i = \frac{r}{m}$ , and the number of payment periods,  $n = mt$ .

Payments may be made **to** the annuity or withdrawn **from** the annuity.

- If you make payments **to** the annuity, you will be concerned with the **Future Value** of the annuity ( $FV$ ).
- If you withdraw payments **from** the annuity, you will be concerned with the **Present Value** of the annuity ( $PV$ ).

Further, payments may be made at the **end** of each payment period or at the **beginning**.

- If payments are made at the **end** of the payment period, it is an **ordinary annuity**.
- If payments are made at the **beginning** of the payment period, it is an **annuity due**.

This gives a total of four different types of annuities and four different formulas to choose from.

**Formulas:**

- **Future Value of an Ordinary Annuity**

$$FV = R \left[ \frac{(1+i)^n - 1}{i} \right]$$

- **Future Value of an Annuity Due**

$$FV = R \left[ \frac{(1+i)^n - 1}{i} \right] (1+i)$$

- **Present Value of an Ordinary Annuity**

$$PV = R \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

- **Present Value of an Annuity Due**

$$PV = R \left[ \frac{1 - (1+i)^{-n}}{i} \right] (1+i)$$

**Examples:**

1. At the end of each month, you place \$100 into an account bearing 6% interest, compounded monthly. What is the balance of the account 5 years after you start?
2. A company establishes a sinking fund to pay a debt of \$100,000 due in 4 years. At the beginning of each six-month period, they deposit  $R$  in an account paying 9%, compounded semi-annually. How big must the payments be to pay the debt on time?
3. Your retirement account earns 7%, compounded quarterly. How much must the account contain when you retire if you want to withdraw \$6000 at the end of each quarter for 30 years?
4. You inherit \$200,000 and invest it at 3%, compounded monthly. If you withdraw \$1000 at the beginning of every month, how long will the money last?