

The equation of a line

1. Given two points: $(x_1, y_1), (x_2, y_2)$.
Compute slope = $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Line equation: $y = m(x - x_1) + y_1$.

Supply-Demand Problems

1. Market equilibrium is at the quantity and price where supply and demand intersect (combine equations!).
2. If price is above the market equilibrium, then there is a surplus.
If price is below, there is a shortage.

Examples directly from old exams:

“The supply function is linear. The supplier is willing to provide 35 items if the price is \$80/item, but only 5 items if the price is \$20/item. Find the supply function.”

“The total cost function is linear. The fixed cost is \$3482 and the total cost to produce 20 Trinkets is \$4004. Find the total cost function.”

“Find the market equilibrium.”

“If the price is \$49/item, is there a surplus or shortage? By how many items is the shortage or surplus?”

Definitions and Functional Notation

$$TR(x) = px$$

$$TC(x) = VC(x) + FC$$

$$P(x) = TR(x) - TC(x)$$

$$AC(x) = \frac{TC(x)}{x}$$

$$AVC(x) = \frac{VC(x)}{x}$$

$$MR(x) = \frac{TR(x + \text{'one'}) - TR(x)}{\text{'one'}}$$

$$MC(x) = \frac{TC(x + \text{'one'}) - TC(x)}{\text{'one'}}$$

$$ATS(t) = \frac{D(t)}{t}$$

Examples directly from old exams:

“If $AVC(q) = 0.01q^2 - 0.5q + 26$ and $FC = 240$,

give the formulas for $TC(q)$ and $AC(q)$.”

“If price is $p = -16q + 1000$,

give the formula for $TR(q)$ and $MR(q)$ ”

“If $TC(q) = 0.01q^2 + q + 150$,

give the formula for $AC(q)$ and $AVC(q)$.”

“If $D(t) = 2.4t - 0.016t^2$

give the formula for $ATS(t)$ and $\frac{D(10+h) - D(10)}{h}$.”

“If $f(x) = x^2 - 10x + 36$,

compute and simplify $\frac{f(8+h) - f(8)}{h}$.”

Find and Interpret the Vertex.

1. Given $y = ax^2 + bx + c$.
If $a > 0$, the parabola opens upward.
If $a < 0$, the parabola opens downward.
2. The vertex of the parabola $y = ax^2 + bx + c$ occurs at $x = -\frac{b}{(2a)}$.
3. Read the question carefully. Is it asking for
... x -coordinate of the vertex?
... value of the function at the vertex?
... some other quantity corresponding to the vertex?

Solve Quadratic Equations

1. To solve $ax^2 + bx + c = 0$ (it must look **exactly** like this).

Use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{(2a)}$$

Examples directly from old exams:

“What is the largest possible total revenue?”

“If $AVC(q) = 0.01q^2 - 0.5q + 26$, what is shutdown price (SDP)?”

“Determine the longest range of quantities on which profit increases.”

“Find the longest range of quantities over which the total revenue increases, but the profit decreases.”

“Find the price that corresponds to maximum profit.”

“Find when $AVC(q) = 0.01q^2 - 0.5q + 26$ is equal to 28.”

“If $TR(q) = -0.5q^2 + 6q$, then find the range of quantities over which total revenue is at least 12 hundred dollars.”

“For what quantities is profit exactly \$3000.”

Remember Standard Applications.

1. Max Profit:

If profit is a quadratic, we can find the vertex. OR We can solve $MR(x) = MC(x)$.

2. Breaking Even Point (Profit = 0):

When $TR(x) = TC(x)$, the profit is zero.

3. Break Even **Price**:

The break even price (BEP) is the lowest y -value of $y = AC(x)$.

Also the y -value when $AC(x) = MC(x)$.

4. Shutdown Price:

The shutdown price (SDP) is the lowest y -value of $y = AVC(x)$.

Also the y -value when $AVC(x) = MC(x)$.

Examples directly from old exams:

“At what quantities do you break even?”

“What selling price leads to the largest possible profit?”

“Compute the shutdown price (SDP).”

Solve a System of Equations.

1. Solve for one variable in one of the equation.
2. Substitute into the other equation.
Simplify and solve for the one remaining variable.
3. Go back and get the other variable.
Check your answer!

Linear Programming.

1. Give constraints and graph overlapping feasible region.
2. Find the corners that are touching the feasible region.
3. Evaluate the objective at these corner.

Examples directly from old exams:

You have to do use these skills in every linear programming problem and every supply/demand problem (so you will use these skills at least once on every second midterm, see archive).

Basics of Exponentials/Logarithms and Solving.

1. Be able to solve equations as we have been discussing in class.
2. In particular be able to solve equations that involve e^x in some way.
3. Isolate the exponential.
Use logarithms and other inverses correctly.
Check your answer!

Several examples from old exams:

“Solve $14400 = 12000e^{3r}$ for r .”

“Solve $200000 = 350000(1 - e^{-0.07t})$ for t .”

“Solve $10(2)^x = 500$ for x .”