

CHAPTER 6 SUMMARY

Lump Sum Accounts: Money is deposited by you at the beginning only.

General Tips:

1. Read the question carefully. What type of account?
2. Convert all times to years (if given in months divide by 12, if given days divide by 365).
3. Draw a timeline to help you understand the question. As I have done in class.
4. The total amount of interest earned is the difference between P and F. That is,

$$\text{Total Interest} = F - P$$

Simple Interest (or anything where the same amount is being ADDED each year):

$$F = P(1 + rt) = P + Prt$$

Note that Pr = 'the amount added each year'.

Algebra Skills: Only divisions and subtraction are needed to solve problems involving simple interest.

Compounding m times a year:

$$F = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$APY = \left[\left(1 + \frac{r}{m}\right)^m - 1\right] \cdot 100 \%$$

Algebra Skills: To solve for P you will just have to *divide*.

To solve for r you will need to use a *root* at some point.

To solve for t you will need to use a *natural logarithm* at some point.

Continuous Compounding:

$$F = Pe^{rt}$$

$$APY = [e^r - 1] \cdot 100 \%$$

Algebra Skills: To solve for P you will just have to *divide*.

To solve for r you will need to use a *natural logarithm* at some point.

To solve for t you will need to use a *natural logarithm* at some point.

Annuity Accounts: There are regular payments or withdrawals.

Two types of Annuities:

Ordinary Annuities: payments made at the END of each compounding period

Annuities Due: payments made at the BEGINNING of each compounding period.

Two types of questions:

Future Value Questions: start with zero dollars in the account, regular deposits, find the future value.

Present Value Questions: start with 'a lot of money' (call this P), make regular withdrawals, end with zero in the account.

First, if you are given r = decimal interest rate, and m = number of compounding periods in a year, then for simplicity (and so we can fit the formulas on the page), we compute:

$$i = \frac{r}{m} = \text{rate at each compounding} \quad \text{and} \quad n = mt = \text{total number of compoundings}$$

	Ordinary (Payments at END of each period)	Due (Payments at BEGINNING of each period)
FV = ?? (so you start with zero)	$F = R \frac{(1+i)^n - 1}{i}$	$F = R \frac{(1+i)^n - 1}{i} (1+i)$
PV = ?? (so you end with zero)	$P = R \frac{1 - (1+i)^{-n}}{i}$	$P = R \frac{1 - (1+i)^{-n}}{i} (1+i)$

Algebra Skills: To solve for R you will just have to *divide*.

To solve for n you will need to use a *natural logarithm* at some point.

Steps to do an annuity problem:

Step 1: Identify the type of account and type of question: Ordinary or Due? FV or PV?

Step 2: Identify the given facts: $r = ??$, $m = ??$, $t = ??$, $R = ??$, FV/PV = ??

Step 3: Compute $i = \frac{r}{m}$ and $n = mt$.

Step 4: Use the correct formula. In computation, compute the number next to R first.

Side note:

$$\begin{aligned} \text{Total Dollar Amount of Payments} &= R \cdot n \\ \text{Total Interest Earned in a FV question} &= F - R \cdot n \end{aligned}$$

Loans: Loans are ordinary annuities, present value questions for the bank (they give you the money, and you pay off the value they gave you with interest). We'll discuss this topic when I lecture on 6.5, here is what we will learn:

$$\begin{aligned} \text{Total Dollar Amount of Payments} &= R \cdot n \\ \text{Total Interest Paid in a PV question} &= R \cdot n - P \end{aligned}$$

After k payments on a loan, the unpaid balance (i.e. payoff amount) is given by

$$\text{Unpaid Balance} = R \frac{1 - (1+i)^{-(n-k)}}{i}$$