Math 111

Solutions for Group Activity: Sequences and Population Growth

1. A bacteria colony doubles in size every hour. At noon, there are 1000 bacteria. Let $B(t)$ be the number of bacteria in the colony t hours after noon.

   (a) How many bacteria are in the colony at 1:00 pm? 2:00 pm? 3:00 pm? 6:00 pm?
   ANSWER:

time	# bacteria	time	# bacteria
1:00	2000	3:00	8000
2:00	4000	6:00	64,000

   (b) Give a formula that gives $B(t)$ as a function of t.
   ANSWER: $\boxed{B(t) = 1000 \cdot 2^t}$

   (c) If $B(t) = 2500$, what is $B(t+1)$? (That is, if you know that the population is 2500 at time t, what will the population be one hour later?)
   ANSWER: $\boxed{5000}$

   (d) If $B(t) = 10,000$, what is $B(t-1)$? (That is, if you know that the population is 10,000 at time t, what was the population one hour earlier?)
   ANSWER: $\boxed{5000}$

   (e) Use your formula from part (b)to answer the following.

   i. What is the population at 12:30 pm? at 1:45 pm? at 8:20 pm? (Round to the nearest bacterium.)
   ANSWER:

time	t	# bacteria
12:30	0.5	$B(0.5) = 1414$
1:45	1.75	$B(1.75) = 3364$
8:20	$8\frac{1}{3} = \frac{25}{3}$	$B(\frac{25}{3}) = 322,540$

   ii. When will the colony contain 1,000,000 bacteria? (How many hours after noon?)
   ANSWER: Find the value of t such that $B(t) = 1,000,000$:

   $$\begin{aligned} 1000000 &= 1000 \cdot 2^t \\ 1000 &= 2^t \\ \ln(1000) &= t\ln(2) \\ t &= \frac{\ln(1000)}{\ln(2)} \approx 9.97 \text{ hours} \end{aligned}$$

   iii. Recall that, if a quantity changes from an OLD value to a NEW value, then the percentage change in the quantity is given by

   $$\frac{\text{NEW} - \text{OLD}}{\text{OLD}} \times 100\%.$$

   What is the percentage change in the population from noon to 12:30 pm?
   ANSWER: OLD=1000 and NEW=1414 (from part (i)). So, percentage change is

   $$\frac{1414 - 1000}{1000} \times 100\% = 41.4\%.$$

2. A second colony increases its population by 75% every 30 minutes. There are 5000 bacteria in this colony at noon.

   (a) How many bacteria are in this colony at 12:30 pm? 1:00 pm? 1:30 pm? 2:00 pm? 3:00 pm?
   ANSWER:

time	# bacteria
12:30	$5000 + 0.75 \cdot 5000 = 5000(1.75) = 8750$
1:00	$8750 + 0.75 \cdot 8750 = 8750(1.75) = 5000(1.75)^2 = 15313$
1:30	$15313(1.75) = 5000(1.75)^3 = 26797$
2:00	$26797(1.75) = 5000(1.75)^4 = 46895$
3:00	$46895(1.75)(1.75) = 5000(1.75)^6 = 143615$

   (b) By what factor must you multiply the population at one specific time to get the population 30 minutes later?
   ANSWER: $\boxed{1.75}$

   (c) By what factor must you multiply the population at one specific time to get the population one hour later?
   ANSWER: $\boxed{(1.75)^2}$

   (d) Let $C(t)$ represent the population of this colony t hours after noon. Give a formula for $C(t)$ as a function of t. (Again, you'll need to relate $C(t)$ to the population at noon and use your answer to part (c) of this question.)
   ANSWER: $\boxed{C(t) = 5000[(1.75)^2]^t}$

   (e) When will this population contain 1,000,000 bacteria?
   ANSWER: Again, set $C(t) = 1000000$ and solve for t:

$$\begin{aligned}
1000000 &= 5000[(1.75)^2]^t \\
200 &= [(1.75)^2]^t \\
\ln(200) &= t\ln(1.75)^2 \\
t &= \frac{\ln(200)}{\ln(1.75)^2} \approx 4.73 \text{ hours}
\end{aligned}$$

   (f) What is the percent change in this population over any one-hour period? (Round to the nearest percent.)
   ANSWER: The percent change will be the same over any one hour period. So we'll compute the percent change from noon to 1:00 pm. Then OLD=5000 and NEW=15313 and percentage change is

$$\frac{15313 - 5000}{5000} \times 100\% = 206\%.$$