

Math 111

Solutions for Group Activity: A Fair Deal

Situation: XYZ Corporation issues promissory notes in \$1000 denominations under the following terms: purchase a \$1000 note now and, two years from now, they will give the owner of the note \$2000. You purchased one of these notes a year ago. It will mature in another year but you need cash now to pay your rent. A friend has offered to buy the note from you. The purpose of this activity is to determine how much your friend should pay for the note.

Define the following values:

- The amount someone pays for the note is their **principal** and will be denoted P .
- The amount someone receives when they sell the note will be denoted A .
- The **return** on the investment will be denoted R . Note $R = A - P$.
- The **rate of return** will be the return R as a percentage of the principal P . The rate of return expressed as a decimal will be denoted p . Then, $p = \frac{R}{P}$ and the rate of return is $p \times 100\%$.

1. **Deal #1:** Your friend offers to pay you \$1000 for the note.

(a) In this scenario, identify the values of P , A , and R for each person: you and your friend.

ANSWER:

	You	Your Friend
P	\$1000	\$1000
A	\$1000	\$2000
R	\$0	\$1000

(b) What is your rate of return in this scenario? Your friend's rate of return?

ANSWER: Your rate of return is 0%. Your friend's is 100%.

(c) Does this deal seem fair? (There isn't necessarily one "correct" answer here. The point is to have a conversation with the members of your group. The numbers you computed in parts (a) and (b) should play a part in your decision.)

ANSWER: I say no. Your money was tied up for a year and you earn nothing. Your friend's money is doubled after one year. While this deal may get you out of a jam, the benefits of this deal are mostly in your friend's favor.

2. **Deal #2:** You say that, since the investment increases by a total of \$1000 during the two years, you should each get a return of \$500.

(a) In this scenario, identify the values of P and A and the **rate** of return for each of you.

ANSWER:

	You	Your Friend
P	\$1000	\$1500
A	\$1500	\$2000
rate of return	50%	33.33%

(b) Does this deal seem fair?

ANSWER: I say no. This deal leans more toward your favor than your friend's. You pay only \$1000 to earn \$500 at the end of one year. Your friend pays \$1500 to earn the same dollar amount at the end of one year.

3. **Deal #3:** You explain the situation to your Math 111 TA who says that the way to make the deal fair is for both of you to earn the same **rate** of return and suggests the following: let X be the amount your friend will pay you for the note and let p be the rate of return that you will both earn, expressed as a decimal.

- (a) In this scenario, you will pay \$1000 and receive X from your friend. The amount of your return should be $(p \times 100)\%$ of your original investment. That is,

$$X = \$1000 + 1000p.$$

Similarly, your friend will pay X and receive \$2000. But the \$2000 should give your friend a return of $(p \times 100)\%$ of his/her investment. That is,

$$\$2000 = X + Xp.$$

You now have two equations in two variables. Solve the system for X and p . (Round p to four digits after the decimal and round X to the nearest cent.)

ANSWER: Substitute $X = 1000(1 + p)$ into the second equation:

$$\begin{aligned} 2000 &= 1000(1 + p)(1 + p) \\ 2000 &= 1000(1 + p)^2 \\ 2 &= (1 + p)^2 \\ \pm\sqrt{2} &= 1 + p \\ p &= -1 \pm \sqrt{2} \end{aligned}$$

Since p must be positive, $p = -1 + \sqrt{2} \approx 0.41421$. Plug into $X = 1000(1 + p)$ to find $X = \$1414.21$.

- (b) In this scenario, what is the return for each of you?

ANSWER: You pay \$1000 and receive \$1414.21. Your return is \$414.21. Your friend pays \$1414.21 and receives \$2000. Your friend's return is \$585.79.

- (c) Does this deal seem fair?

ANSWER: I say yes. Your friend puts in a little more than you and gets a little more in return but you each earn 41.421% of your original investment.

- (d) Which one of the three deals would you choose? Or is there another scheme that seems fairer to you than any of these. If so, identify the principal, return, and rate of return for each of you and explain why it appeals to you.

ANSWER: Answers may vary.