Name: ______________________________ Section: __________

Math 111

Group Activity: The Method of Linear Programming

The *Grass is Greener* lawn care company produces two different lawn fertilizers: Regular and Deluxe. The profit on each bag of Regular is \$0.75, while the profit on each bag of Deluxe is \$1.20. Each bag of Regular contains 3 pounds of active ingredients and 7 pounds of inert ingredients. In contrast, each bag of Deluxe contains 4 pounds of active ingredients and 6 pounds of inert ingredients. Due to limited warehouse facilities, the company can stock up to 8,400 pounds of active ingredients and 14,100 pounds of inert ingredients.

Let x denote the number of bags of Regular fertilizer and y denote the number of bags of Deluxe fertilizer the company will produce.

In this activity, you'll use the method of linear programming to find the amount of each fertilizer the company should produce in order to maximize profit and investigate why this method gives the optimal solution.

1. Linear programming is a method for maximizing or minimizing an objective function subject to one or more constraints. In this problem, we wish to maximize profit. Find a formula $P(x, y)$ that gives the amount of profit the company will earn by selling x bags of Regular fertilizer and y bags of Deluxe fertilizer. This is the objective function.

   ANSWER: $\boxed{P(x, y) = 0.75x + 1.20y}$

2. A constraint in a linear programming problem can be expressed as an inequality (a statement involving the symbol "$\leq$" or "$\geq$".) In this problem, two of the constraints are simply

   $$x \geq 0 \text{ and } y \geq 0.$$

   To get the remaining constraints, note that production is limited by the amounts of active ingredients and inert ingredients that are available.

   - Determine the number of pounds of **active ingredients** needed to produce x bags of Regular fertilizer and y bags of Deluxe fertilizer and write an inequality that expresses the constraint due to the limitations on **active ingredients**.
     ANSWER: $\boxed{3x + 4y \leq 8400}$
   - Determine the number of pounds of **inert ingredients** needed to produce x bags of Regular fertilizer and y bags of Deluxe fertilizer and write an inequality that expresses the constraint due to the limitations on **inert ingredients**.
     ANSWER: $\boxed{7x + 6y \leq 14100}$

   You should now have four constraints.

3. Sketch and shade the system of inequalities representing your four constraints. This gives the feasible region for this problem: the set of points (x, y) that correspond to values of x and y that you can feasibly produce. Find the coordinates of all of the corners of the feasible region.

   ANSWER: Corners of feasible region: $\boxed{(0, 0),\ (600, 1650),\ (0, 2100), \text{ and } (2014.29, 0)}$

4. The maximum value of the objective function (profit, in this case) must occur at one of the corners of the feasible region. (We'll see why later in this activity.) This means that we can find the maximum value of the objective function by evaluating it at the corners and choosing the largest of those values. Evaluate the profit function from #1 at each of the corners you found in #3.

   ANSWER: $\boxed{P(0, 0) = \$0,\ P(600, 1650) = \$2430,\ P(0, 2100) = \$2520, \text{ and } P(2014.79, 0) = \$1511.09}$

   How many bags of each type of fertilizer should you make to maximize profit?

   ANSWER: $\boxed{\text{0 bags of Regular and 2100 bags of Deluxe}}$

   What is the maximum possible profit?

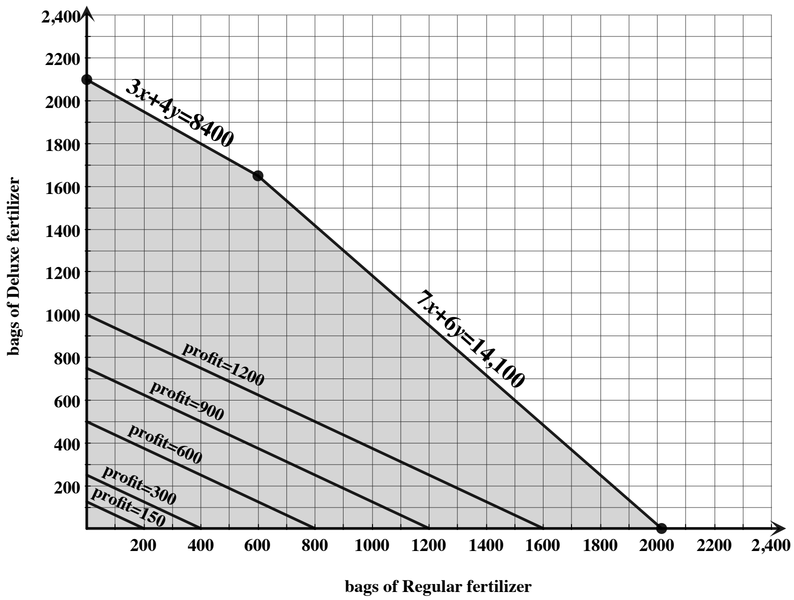
   ANSWER: $\boxed{\$2520}$

5. Now, we'll investigate why maximum profit occurs at one of the corners of the feasible region. The feasible region is graphed below. (Check that it's the same as the one you drew earlier.)

   (a) Recall that profit is given by the function $P(x, y) = 0.75x + 1.20y$. Let's look at the amounts of each type of fertilizer we'd need to make in order to have a profit of exactly \$150. Those would be the points (x, y) that satisfy the equation

   $$0.75x + 1.20y = 150.$$

   This is a line. Find its x-intercept and y-intercept and verify that I've drawn this line on the graph of the feasible region below.



   (b) Repeat this process several times until you understand and can describe the behavior of these constant profit lines: draw the lines in the feasible region above that show the values of x and y that give a profit of \$300, \$600, \$900, and \$1200. What features do these lines share? What happens as you consider larger and larger profit?

   ANSWER: These lines are all parallel—they have the same slope. As profit gets larger, the lines get farther from the origin.

   (c) Explain why the maximum profit must occur at one of the corners of the feasible region. (Hint: Look at how the constant profit lines change as profit increases and remember that only points in the feasible region matter.)

   ANSWER: The largest possible profit will correspond to the constant profit line (parallel to the ones in the picture above) that is farthest from the origin but that still contains at least one point of the feasible region. If you think of these profit lines as moving through the feasible region, the last point in the feasible region that will get "hit" by one of these lines must be a corner.

6. Finally, suppose you must make **at least** 400 bags of the Regular fertilizer and **at least** 600 bags of the Deluxe and you adjust your costs so that you earn a profit of \$1.00 for each bag of Regular. You continue to earn \$1.20 per bag of Deluxe. This means that your new objective function is

   $$P(x, y) = 1.00x + 1.20y.$$

   Your new constraints are:

   $$3x + 4y \leq 8400,\ 7x + 6y \leq 14100,\ x \geq 400 \text{ and } y \geq 600.$$

   Complete the feasible region on the graph below, find all the corners, and determine the maximum possible profit and the production levels that yield maximum profit.

   SOLUTION: The corners of the new feasible region are $(400, 600)$, $(400, 1800)$, $(600, 1650)$, and $(1500, 600)$. Maximum profit is \$2580. To maximize profit, sell 600 bags of Regular and 1650 bags of Deluxe.