Name: _____ Section: _____

Math 111

Solutions for Group Activity: Maximizing Profit Three Ways

You work for Tasty Tours, organizing tours of a local winery. In order to make your tour competitive with other companies, you offer price breaks for larger groups. The following table gives the price per person (in dollars), the total revenue (in dollars), and the marginal revenue (in dollars per person) for different values of q (in number of people).

q	10	20	25	30	35	40	60
p	26	22	20	18	16	14	6
TR	260	440	500	480	560	560	360
MR	21.60	13.60	9.60	5.60	1.60	−2.40	−18.40

Your costs come from the winery and caterer, who charge you \$8 per person. You also have fixed costs in the amount of $\$FC$.

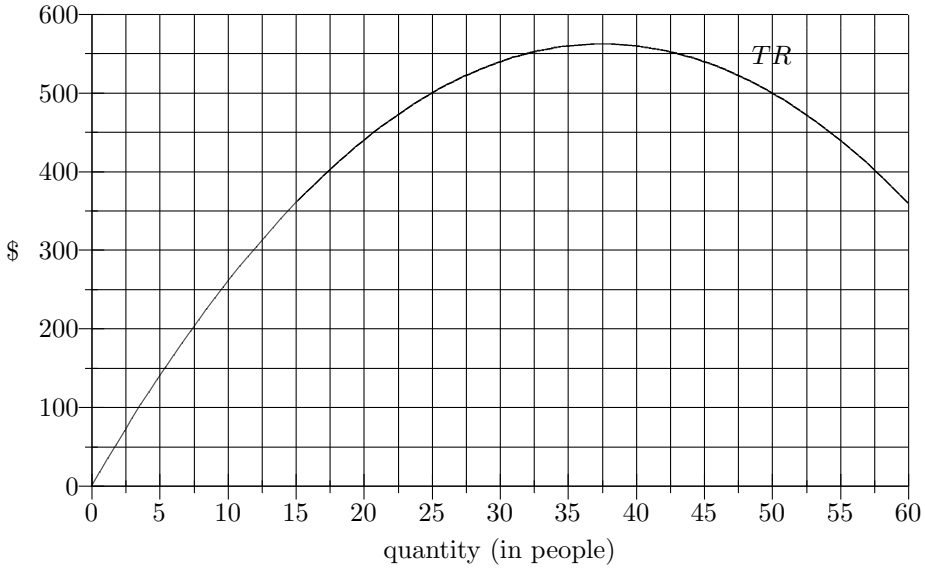
1. (a) The value of marginal cost is the same for all quantities. What is the marginal cost? Include units.

   ANSWER: Marginal cost is \$8 per person.

   (b) Recall how marginal revenue and marginal cost determine the quantity that maximizes profit and use the above table to estimate the number of people that yields maximum profit.

   ANSWER: Profit is maximized at a quantity at which $MR = MC$. Since MC is always \$8 per person and, according to the values in the table, MR seems to be decreasing, it's reasonable to conclude that marginal revenue will equal marginal cost at a quantity between $q = 25$ and $q = 30$. Since \$8 is *almost* half-way between \$9.60 ($MR(25)$) and \$5.60 ($MR(30)$), I would estimate that profit is maximized at $q \approx 27$ people.

2. The following is the graph of total revenue.



   (a) Sketch the graph of variable cost on the axes above.

   (b) Use the graphs of TR and VC to approximate the number of people that yields maximum profit.

   ANSWER: The graph of VC is a diagonal line with slope 8. Use the sliding ruler method to find the quantity at which $MR = MC$. This should be at $q \approx 27$ people.

3. (a) The price per person p is a linear function of quantity q. Using the information given in the table, find this linear function.

ANSWER: The price function goes through the points $(10, 26)$ and $(20, 22)$. (You could use two other points from the table instead.) The equation of the line through these points is:

$$\boxed{p = -0.4q + 30.}$$

(b) You now have a formula for the price of a tour per person for a group of q people. Use this to find the formula for your total revenue: $TR(q)$. What are the units associated with q in your formula? With $TR(q)$?

ANSWER: If you sell q tickets at p dollars per person, your total revenue is $q \times p$. So, $TR(q) = q(-0.4q + 30)$. Distributing the q through gives:

$$\boxed{TR(q) = -0.4q^2 + 30q.}$$

Here, q is measured in *people* and TR is in *dollars*.

(c) Recall that $MR(q) = \dfrac{TR(q+1) - TR(q)}{1}$ and find a linear formula for $MR(q)$. What are the units associated with q in your formula? With MR(q)?

ANSWER:

$$TR(q+1) = -0.4(q+1)^2 + 30(q+1) = -0.4(q^2+2q+1) + 30(q+1) = -0.4q^2 - 0.8q - 0.4 + 30q + 30.$$

$$TR(q) = -0.4q^2 + 30q$$

$$TR(q+1) - TR(q) = -0.8q + 29.6$$

$$\frac{TR(q+1) - TR(q)}{1} = -0.8q + 29.6.$$

$$\boxed{MR(q) = -0.8q + 29.6.}$$

Again, q is measured in *people*. MR is measured in *dollars per person.*

(d) Use the *formulas* for MR and MC to determine the number of people that yields maximum profit.

ANSWER: Set $MR(q) = MC(q)$ and solve for q:

$$-0.8q + 29.6 = 8 \Rightarrow \boxed{q = 27.}$$

4. You used three different methods to find the quantity that maximizes profit: using a table of values of MR, graphs of TR and VC, and using formulas for MR and MC. The three methods should have yielded similar results. Discuss the advantages and disadvantages of using each method.

ANSWER: The first two methods were reasonably quick and easy but gave approximations. The third method had many steps but gave an exact answer rather than an approximation. Can you think of other advantages or disadvantages for any of the methods?