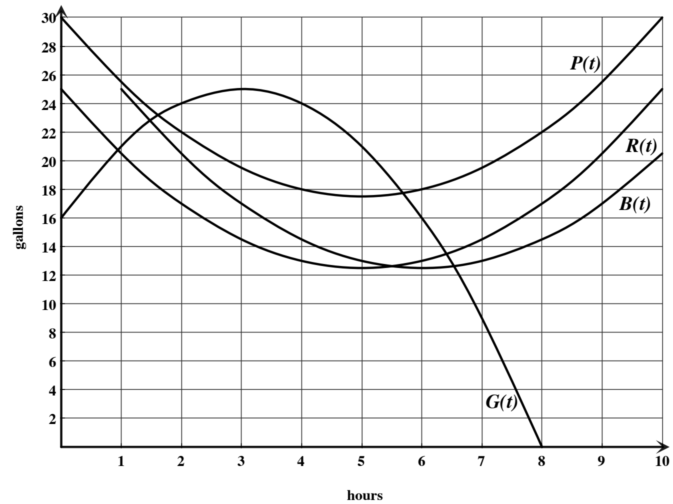
Math 111

Solutions for Group Activity: Two Vats of Water

Water is flowing in and out of two vats, a red vat and a green vat. The graphs to the right show the amounts of water in the two vats over a several-hour period. We also have a gauge which measures the difference:

Amount in Green Vat
minus
Amount in Red Vat.

Note that the gauge starts out negative, is then positive for several hours, and again becomes negative.



1. Use the graphs to find approximate answers to the following.
   (a) Find the time at which the water in the red vat hits its lowest level.
   ANSWER: Find the t-coordinate of the vertex of $R(t)$: $\boxed{t \approx 5 \text{ hours}}$
   (b) Find the lowest value of the water level in the red vat.
   ANSWER: Find the "y"-coordinate of the vertex of $R(t)$: $\boxed{12.5 \text{ gallons}}$
   (c) Find the first time at which the two vats contain the same amount of water.
   ANSWER: Find the first time at which the graphs intersect: $\boxed{t \approx 0.9 \text{ hours}}$
   (d) Find the time at which the difference gauge reaches its highest value.
   ANSWER: Find the time at which the graphs are farthest apart vertically: $\boxed{t \approx 3.75 \text{ hours}}$
   (e) Find the highest value the difference gauge reaches.
   ANSWER: Find the largest vertical distance between the graphs: $\boxed{\sim 11 \text{ gallons}}$

2. The formula for the amount of water in the red vat is

$$R(t) = 0.5t^2 - 5t + 25;$$

the formula for the amount of water in the green vat is

$$G(t) = -t^2 + 6t + 16.$$

We'll use algebra and these formulas to answer the same questions. For each, you should compare your answer to the responses you found using the graphs in #1.

   (a) Find the time at which the water in the red vat hits its lowest level.
   ANSWER: Find the t-coordinate of the vertex of $R(t)$: $\boxed{t = 5 \text{ hours}}$
   (b) Find the lowest value of the water level in the red vat.
   ANSWER: Find the "y"-coordinate of the vertex of $R(t)$: $\boxed{R(5) = 12.5 \text{ gallons}}$
   (c) Find the first time at which the two vats contain the same amount of water.
   ANSWER: Find the first time at which $R(t) = G(t)$: $\boxed{t = 0.94 \text{ hours}}$
   (d) To answer part (d), we need a formula for the *difference*, $D(t)$. We said that our difference gauge measures the amount in the green vat minus the amount in the red vat. So,

$$D(t) = G(t) - R(t).$$

   We have formulas for $G(t)$ and $R(t)$. Find a formula for $D(t)$ and use it to find the time at which the difference gauge reaches its highest value.
   ANSWER: Find the t-coordinate of the vertex of $D(t)$: $\boxed{t = \frac{11}{3} \approx 3.67 \text{ hours}}$

(e) Find the highest value the difference gauge reaches.

ANSWER: Find the "y"-coordinate of the vertex of $D(t)$: $\boxed{D(3.67) = 11.17 \text{ gallons}}$

3. Suppose there is a purple vat that always contains exactly 5 gallons more than the red vat.

(a) Sketch a rough graph of the amount in the purple vat on the graph at the beginning of this Activity.

(b) Give a formula for $P(t)$, the amount of water in the purple vat after t hours.

ANSWER: $\boxed{P(t) = R(t) + 5 = 0.5t^2 - 5t + 30}$

(c) Answer each of the following as accurately and efficiently as possible. For several of these questions, you should be able to use your answers to #1 and/or 2 and the new graph of $P(t)$ to answer without doing any calculations. For some, however, you must do some calculations to get the exact answer.

i. Find the time at which the water in the purple vat hits its lowest level.
ANSWER: The purple vat and the red vat reach their lowest level at the same time: $\boxed{t = 5 \text{ hours}}$

ii. Find the lowest value of the water level in the purple vat.
ANSWER: The lowest level of the purple vat is 5 gallons more than the lowest level of the red vat: $\boxed{17.5 \text{ gallons}}$

iii. Find the first time at which the water levels in the **purple** and **green** vats are the same.
ANSWER: We have to compute this one. Set $P(t) = G(t)$ and solve the resulting equation: $\boxed{t = 1.64 \text{ hours}}$

iv. If there is a difference gauge that measures the amount in the green vat minus the amount in the purple vat, find the time at which this difference gauge reaches its highest value.
ANSWER: The new difference gauge will be highest at the same time as the old gauge: $\boxed{t = \frac{11}{3} \approx 3.67}$

v. Find the highest value this new difference gauge reaches.
ANSWER: The greatest vertical distance between the graphs of $P(t)$ and $G(t)$ is 5 gallons smaller than the distance between $R(t)$ and $G(t)$: $\boxed{6.17 \text{ gallons}}$

4. Suppose there is a blue vat whose water level is described by the following: At time t, the blue vat contains the same amount that the red vat contained one hour earlier.

(a) Sketch the amount in the blue vat on the graph at the beginning of the activity. Start at $t = 1$.

(b) Let $B(t)$ be the amount in the blue vat at time t. At time t, the blue vat contains the amount that the red vat contained at $t - 1$. This means that $B(t) = R(t - 1)$. We obtain $R(t - 1)$ by replacing every t in the formula for $R(t)$ with a $t - 1$:

$$B(t) = R(t - 1) = 0.5(t - 1)^2 - 5(t - 1) + 25.$$

Simplify this formula to get a quadratic expression for $B(t)$.

ANSWER: $\boxed{B(t) = 0.5t^2 - 6t + 30.5}$

(c) Answer each of the following as accurately and efficiently as possible. For several of these questions, you should be able to use your answers to #1 and/or 2 and the new graph of $B(t)$ to answer without doing any calculations. For some, however, you must do some calculations to get the exact answer.

i. Find the time at which the water in the blue vat hits its lowest level.
ANSWER: Blue's lowest level will happen one hour later than red's: $\boxed{t = 6 \text{ hours}}$

ii. Find the lowest value of the water level in the blue vat.
ANSWER: This is the same as the lowest water level in the red vat: $\boxed{12.5 \text{ gallons}}$

iii. Find the first time at which the water levels in the **blue** and **green** vats are the same.
ANSWER: We have to compute this one. Set $B(t) = G(t)$ and solve the resulting equation: $\boxed{\text{t=1.48 hours}}$

iv. If there is a difference gauge that measures the amount in the green vat minus the amount in the blue vat, find the time at which this difference gauge reaches its highest value.
ANSWER: We have to compute this one. Find a formula for $F(t) = G(t) - B(t)$ and simplify. Find the t-coordinate of its vertex: $\boxed{t = 4 \text{ hours}}$

v. Find the highest value this new difference gauge reaches.
ANSWER: Compute $F(4)$: $\boxed{9.5 \text{ gallons}}$