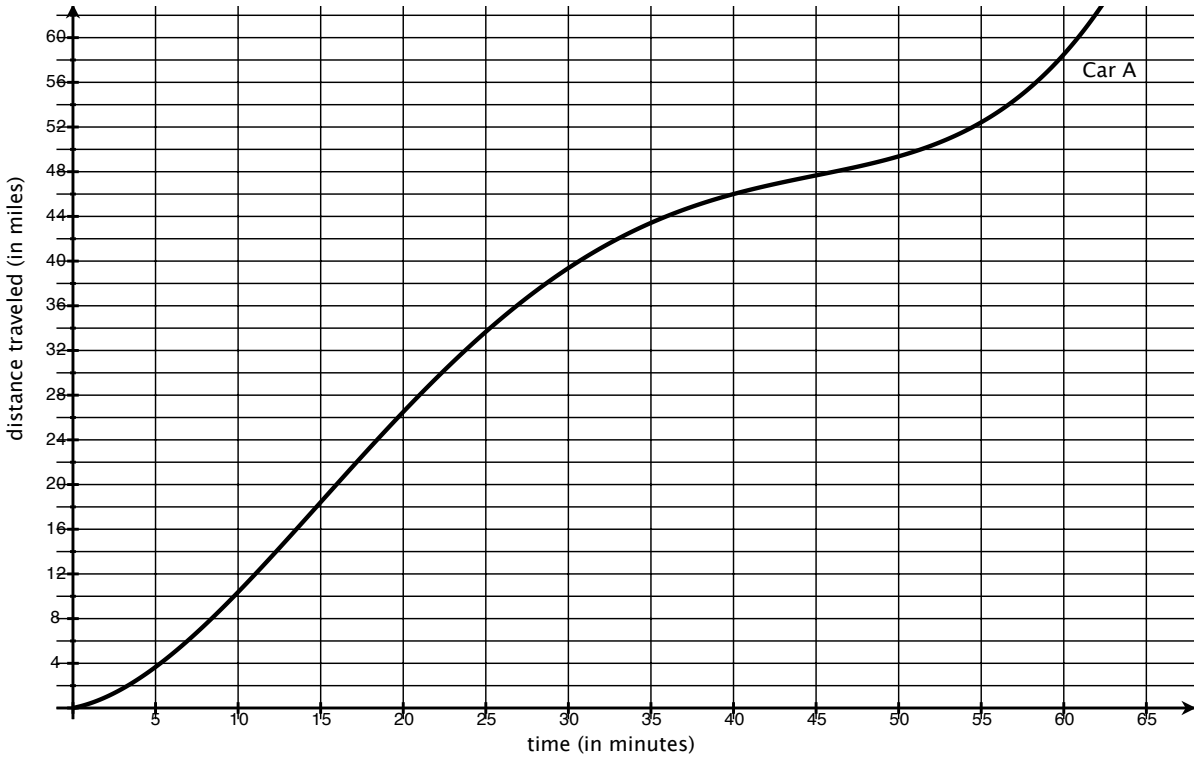
Math 111

Solutions for Group Activity: More Rates of Change

1. The following is the graph of distance traveled versus time for Car A.



All answers are approximations. Your answers may not be exactly the same as mine.

(a) Compute Car A's average speed during the 10-minute interval beginning at $t = 40$ minutes.

ANSWER: Draw the secant line through distance traveled at $t = 40$ and $t = 50$, select two points on your line, and compute the slope. Your answer should be close to $\boxed{\text{0.35 miles per minute}}$.

(b) Find the highest value of Car A's average trip speed. (HINT: Think about the graphical interpretation of average trip speed.)

ANSWER: The graphical interpretation of average trip speed at time t is the slope of a diagonal line through distance traveled at time t. You need to find the slope of the steepest diagonal line that intersects the distance traveled graph (at a second point, in addition to at the origin). Draw the steepest such diagonal line, select two points on your line (one should be $(0, 0)$), and compute its slope. The car's highest average trip speed is approximately $\boxed{\text{1.35 miles per minute}}$.

(c) A second car, Car B, is next to Car A at $t = 0$ and travels 10 miles every 10 minutes. Give the longest time interval during which Car A is ahead of Car B. ((HINT: Draw the graph of Car B's distance traveled on the axes above.)

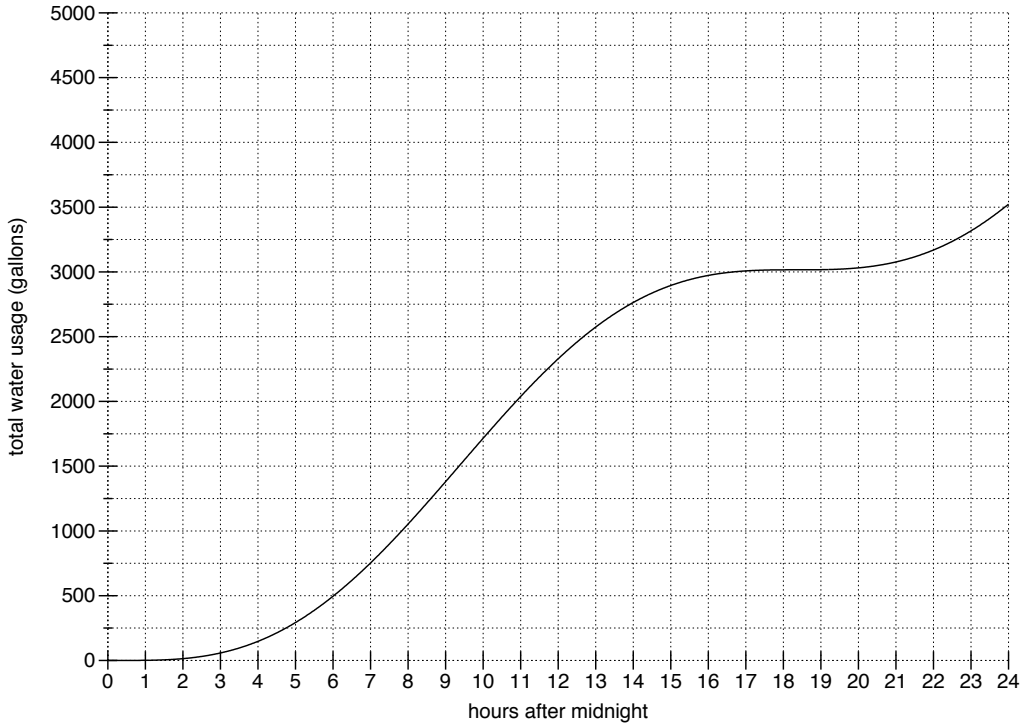
ANSWER: The graph of Car B's distance traveled will be a line that goes through the origin (why?) with slope 1 (why?). Find the interval on which Car A's distance graph is *above* Car B's. Car A is ahead of Car B $\boxed{\text{from about } t = 9 \text{ minutes to about } t = 48 \text{ minutes}}$.

(d) Give a 5-minute interval during which both cars have the same average speed and Car B is ahead of Car A.

ANSWER: Car B's average speed is 1 mpm, the slope of its distance graph. You need to find a five-minute interval on which the secant line through Car A's distance graph has that slope. Place your ruler along Car B's distance graph and slide your ruler, keeping it parallel to Car B, until it goes through Car A's distance graph at two points that are five minutes apart.

$\boxed{\text{There are two possible intervals: from about } t = 1 \text{ to } t = 6 \text{ and from about } t = 54 \text{ to } t = 59}$.

2. A town is using water from a reservoir that is being refilled with a system of aqueducts. The graph below shows the total water drawn from the reservoir over the course of a day, starting at midnight.



(a) Suppose the reservoir is empty at midnight and is filled by the aqueduct at a constant rate of 150 gallons per hour. Sketch a graph of the amount of water that has flowed into the reservoir on the above graph and use it to answer these questions: Will there be enough water in the reservoir to provide for the town during this 24-hour period? If not, when will there be a shortage? If the rate of flow must remain 150 gallons per hour, how much water must be in the reservoir at midnight to avoid a shortage? (HINT: How would you change the inflow graph so that there is never a shortage?)

ANSWER: The graph of inflow is a line through the origin (why?) with slope 150. In order for there never to be a shortage, the inflow graph would have to be above the usage graph at all times. However, there is an interval on which the usage graph is above the inflow graph: there is a shortage from about 8.8 to 20.4 hours. If the rate of inflow remains 150 gallons per hour, the graph of inflow is always a line with slope 150. Place your ruler along the original inflow graph and slide it up, keeping it parallel, until it is at or above the usage graph at all times. This gives a new inflow graph whose y-intercept is the amount that we need to start with at midnight: the town needs approximately 750 gallons in the reservoir at midnight to avoid a shortage.

(b) Suppose instead that the reservoir must be empty at midnight. Again, water flows into the reservoir at a constant rate. What is the smallest that rate could be to avoid a water shortage during this 24-hour period? (HINT: If water flows in at a constant rate, what does the graph of inflow look like? What feature of the inflow graph is represented by the rate of flow in? What must the inflow graph look like if there is never to be a shortage?)

ANSWER: This is very similar to #1(b): draw the steepest diagonal line that intersects the usage graph and compute its slope. The smallest such rate is approximately 197 gallons per hour.

(c) Suppose instead that there are 3000 gallons of water in the reservoir at midnight and that water flows in at a constant rate. What is the smallest that rate could be to avoid a water shortage during this 24-hour period?

ANSWER: The inflow graph on this one will have a y-intercept of 3000. Draw the least steep line with this y-intercept that intersects the usage graph. Find its slope. The smallest such rate is 20.83 gallons per hour.