Math 111 Week 5 Review

This review is not all inclusive. You are expected to know how to do all the problems in the homework.

Worksheet 10: Equations for Lines and Breaking Even

1. The quantity where a company breaks even is the quantity where PROFIT = $0, i.e. \( TR(q) = TC(q) \). (that is, when \( TR \) and \( TC \) intersect).

2. If market price, \( p \), is given, then \( TR(q) \) is a straight line with SLOPE = \( p \) and Y-INTERCEPT = 0. So \( TR(q) = pq \).

3. If fixed cost, \( FC = c \), is given and marginal cost, \( MC = m \), is constant, then \( TC(q) \) is a straight line with SLOPE = \( m \) and Y-INTERCEPT = \( c \). So \( TC(q) = mq + c \).

4. To find the quantity at which the company breaks even:
   (a) Find the equations for \( TR(q) \) and \( TC(q) \).
   (b) Solve when the equations are equal (i.e. when \( TR(q) = TC(q) \)).

5. In general for straight lines, we found that the solution is \( q = \frac{c}{p-m} \).
   (a) If \( q > \) break even quantity, then profit is positive (a gain).
   (b) If \( q = \) break even quantity, then profit is zero (break even).
   (c) If \( q < \) break even quantity, then profit if negative (a loss).

Worksheet 11: Distance, Speed, and Algebra

1. Remember how functional notation works, you need to be very comfortable with functional notation. Here are several random examples:
   • If \( D(t) = 4t - 5t^2 \), then
     (a) \( D(t + 2) = 4(t + 2) - 5(t + 2)^2 \).
     (b) \( D(10t) = 4(10t) - 5(10t)^2 \).
     (c) \( D(w + t) = 4(w + t) - 5(w + t)^2 \).
     (d) \( D(t + 4) - D(t - 1) = [4(t + 4) - 5(t + 4)^2] - [4(t - 1) - 5(t - 1)^2] \).

2. We now can combine our translation skill from WS 1-9 with actual equations. Here are some examples from distance and speed:
   Assume the distance formula for a car is \( D(t) = 4t - 3t^2 \) and consider the following:
   (a) The average trip speed (ATS) is \( \frac{D(t)}{t} \). So
      \[
      ATS(t) = \frac{D(t)}{t} = \frac{(4t - 3t^2)}{t} = \frac{4 - 3t}{t} = 4 - 3t,
      \]
      so the simplified formula is \( ATS(t) = 4 - 3t \).
   (b) The average speed (AS) from \( t \) to five-minutes later is \( \frac{D(t+5) - D(t)}{(t+5) - t} \). So (showing all steps)
      \[
      AS(t) = \frac{D(t+5) - D(t)}{5} = \frac{[4(t + 5) - 3(t + 5)^2] - [4t - 3t^2]}{5}
      \]
      \[
      = \frac{[4t + 20 - 3(t^2 + 10t + 25)] - [4t - 3t^2]}{5}
      \]
      \[
      = \frac{[4t + 20 - 3t^2 - 30t - 75] - [4t - 3t^2]}{5}
      \]
      \[
      = \frac{[-3t^2 - 26t - 55] - [4t - 3t^2]}{5} = \frac{-3t^2 - 26t - 55 - 4t + 3t^2}{5}
      \]
      \[
      = \frac{-30t - 55}{5}.
      \]
      The simplified formula for average speed is \( AS(t) = -6t - 11 \).
3. As you can see, this sometimes takes a little patience, but the payoff is that we get a simple formula that we can then use to answer questions. Using the same example formulas from above consider the questions:

(a) Question: “Find the ATS at $t = 4$.”
Answer: Plug in $t = 4$ to the ATS formula. $ATS(4) = 4 - 3(4) = -8$

(b) Question: “Find the time when ATS is 2.”
Answer: Solve $ATS(t) = 2$. That is, solve $4 - 3t = 2$ which gives $-3t = -2$, so $t = 2/3$.

(c) Question: “Find the AS from $t = 10$ to five-minutes later.”
Answer: Plug in $t = 10$ to the AS formula. $AS(10) = -6(10) - 11 = -71$.

(d) Question: “Find the time such that the AS over the next five-minutes is 3.”
Answer: Solve $AS(t) = 3$. That is, solve $-6t - 11 = 3$. Adding 11 to both sides gives $-6t = 14$. Thus, $t = -6/14 = -3/7$

Worksheet 12: TR/TC, MR/MC, AR/AC and Algebra

1. This is similar to WS 11, but for MR/MC and AR/AC. For example, if $TR(q) = -q^2 + 30q$ and $TC(q) = 15q + 100$, then

(a) The marginal revenue is the change in revenue from $q$ to $q + 1$. So

$$MR(q) = TR(q + 1) - TR(q) = -[(q + 1)^2 + 30(q + 1)] - [-q^2 + 30q]$$
$$= -(q^2 + 2q + 1 + 30q + 30) - [-q^2 + 30q] = -q^2 - 2q - 1 + 30q + 30 + q^2 - 30q$$
$$= -32q + 29.$$  

The simplified formula is $MR(q) = -32q + 29$.

(b) The marginal cost is the change in cost from $q$ to $q + 1$. So

$$MC(q) = TC(q + 1) - TC(q) = [15(q + 1) + 100] - [15q + 100]$$
$$= [15q + 15 + 100] - [15q + 100] = 15q + 115 - 15q - 100$$
$$= 15.$$  

The simplified formula is $MC(q) = 15$.

(c) The average revenue is $\frac{TR(q)}{q}$ so

$$AR(q) = \frac{TR(q)}{q} = \frac{(-q^2 + 30q)}{q} = \frac{(-q + 30)q}{q} = -q + 30.$$  

The simplified formula is $AR(q) = -q + 30$.

(d) The average cost is $\frac{TC(q)}{q}$ so

$$AC(q) = \frac{TC(q)}{q} = \frac{(15q + 100)}{q}.$$  

The simplified formula is $AC(q) = \frac{(15q + 100)}{q}$.

2. With the simplified formulas you can quickly answer questions about this business situation. For example:

(a) To find the quantity that gives max profit use the simplified formulas for $MR$ and $MC$ and solve when $MR(q) = MC(q)$. (Round your answer up).

(b) To find the BEP, solve $AC(q) = MC(q)$, then plug the quantify back into $AC(q)$ to get the height of $AC$.

Theme of WS 10, 11, 12

In general, when given a formula for distance, revenue, or a reservoir, if you are asked to find ATS, AS, MR, MC, etc., then do the follow:

1. Translate the question to functional notation.
2. Replace the functional notation using the function definition.
3. Solve the problem.