Sections 1.1 to 1.3 Review

Algebra Skills Review: Here is a review of a few of the basic skils we discussed.

- Simplifying Fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Here are several examples:

$$(1)\quad 3 \cdot \frac{5x}{2} = \frac{15x}{2} \qquad (2)\quad \frac{2}{5} \cdot \frac{x+3}{7} = \frac{2x+6}{35} \qquad (3)\quad \frac{2}{3}\left(3x + \frac{1}{2}\right) = 2x + \frac{1}{3}$$

- Distribution: $a(b+c) = ab + ac$ and $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$. Here are several examples:

$$(1)\quad 5\left(4 + \frac{x}{5}\right) = 20 + x \qquad (2)\quad \frac{x+5}{5} = \frac{x}{5} + 1 \qquad (3)\quad \frac{3+4x}{5} = \frac{3}{5} + \frac{4x}{5}$$

- To solve an equation:

  1. Expand/Simplify and Clear Denominators.
  2. Get all x's to one side, everything else to the other side.

  Here is a basic example:

$\frac{1}{3}(x-6)$	$= 1 + \frac{x}{5}$	
$x - 6$	$= 3 + \frac{3x}{5}$	multiplied both sides by 3
$5x - 30$	$= 15 + 3x$	multiplied both sides by 5
$5x$	$= 45 + 3x$	added 30 to both sides
$2x$	$= 45$	subtracted $3x$ from both sides
x	$= \frac{45}{2}$	divided both sides by 2

- Inequalities: All the same rules as equal signs, except you change the direction of the inequality if you multiply or divide by a negative number. Here's a basic example:

$\frac{x}{4} - 3$	$< 10 + x$	
$x - 12$	$< 40 + 4x$	multiplied both sides by 4
x	$< 52 + 4x$	added 12 to both sides
$-3x$	< 52	subtracted $4x$ from both sides
x	$> -\frac{52}{3}$	divided by -3 (flip inequality!)

Linear Function Skill Review:

1. A *linear function* can be written in the slope-intercept form

$$y = mx + b \qquad \text{(or, equivalently, } f(x) = mx + b\text{)}$$

   where

   - m = slope $= \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are *any* two points on the line.
   - $b = y$-intercept = 'y-value when $x = 0$'

2. The *general form* of a linear function is:

$$ax + by = c.$$

   We can solve for y in terms of x to give an equivalent line in the slope-intercept form, but often there is no need. If we want to graph a line in this form it is easy to find the x and y intercepts. To find the x-intercept, let $y = 0$ and solve for x. And to find the y-intercepts, let $x = 0$ and solve for y. Then you have two points and it is easy to draw the line.

3. The *point-slope* form is perhaps the quickest to use if you are trying to give the equation for a line. Given any two points, (x_1, y_1) and (x_2, y_2), on a line we can quickly give the equation of the line by finding the slope, then writing

$$f(x) = m(x - x_1) + y_1 \qquad \text{(or, equivalently, } f(x) = m(x - x_2) + y_2\text{)}$$

4. Other notable facts:

   - Two lines are parallel if they have the same slope.
   - Two lines are perpendicular if their slopes are negative reciprocals of each other (That is, $m_2 = -\frac{1}{m_1}$).
   - $y = c$ describes the *horizontal* line of all points where $y = c$.
   - $x = d$ describes the *vertical* line of all points where $x = d$.

Problem Set Up Review:

- I suggested the following problem solving strategies:

  1. Read the question carefully: What are you given? What do you want? What are the units?
  2. Label everything! Introduce variables for unknown values. (This is often the key step you need to get started on the problem.)
  3. Translate to algebraic expressions.
  4. Solve for anything you can. Even if you cant immediately answer the question, solve for something related.
  5. Make sure your final answer actually answers the original question.

  If you're ever stuck, make up numbers and work out an example!

- Here is an example from an old exam (just to illustrate this problem solving method):
  Bob has \$50,000 to invest (he will invest it all). He has chosen one relatively safe investment fund that has an annual yield of 5% and another, riskier fund that has a 11.5% annual yield. How much should he invest in each fund if he would like to earn \$3,000 per year in interest from his investments?

  Answer:

  1. Read the question. Your final answer will be two dollar amounts (how much to put into each account).
  2. Let $x =$ 'the amount you put into the 5% account' and
     let $y =$ 'the amount you put into the 11.5% account.'
  3. The total amount of money invested is $x + y$ and it needs to equal \$50,000. Thus

     $$x + y = 50000, \qquad \text{which means } y = 50000 - x$$

     The interest from the first account will be $0.05x$ and the interest from the second will be $0.115y$ and we want the interest to total to \$3,000. Thus,

     $$0.05x + 0.115y = 3000.$$

     Combining these two facts gives

     $$0.05x + 0.115(50000 - x) = 3000.$$

  4. Now we solve:

$0.05x + 0.115(50000 - x)$	$= 3000$	
$0.05x + 5750 - 0.115x$	$= 3000$	expanded
$0.05x - 0.115x$	$= -2750$	subtracted 5750 from both sides
$-0.065x$	$= -2750$	combined like terms
x	$= \frac{-2750}{-0.065} = \$42307.69$	divided by -0.065

  5. Therefore, we need to invest $x = \$42,307.69$ into the 5% account and $y = \$7,692.31$ into the 11% account.