Chapter 6 Flowchart

Here is a 'flowchart' that should help you organize your thinking about interest bearing accounts. The following pages also contain 17 practice problems and solutions!

If you realize you are dealing with an account that earns interest, then ask yourself the following questions:

1. Are there regular payments (i.e. regular deposits or withdrawls)?
   - If YES, then it is a ANNUITY. You will use one of the four annuity formulas.
   - If NO, then you are doing a LUMP SUM problem. Any time money is left to sit in an interest bearing account, then it is a LUMP SUM problem. You will use one of the lump sum formulas.
2. If it is a LUMP SUM problem, then it is very easy to determine the formula because the problem has to tell you outright.
   - If it says "simple interest", then it is simple interest: $F = P(1 + rt)$.
   - If it says "continuous compounding", then it is continuous compounding: $F = Pe^{rt}$.
   - If it says "compounded m times a year", then it is discrete compounding: $F = P\left(1 + \frac{r}{m}\right)^{mt}$.
3. If it is an ANNUITY problem, then is also easy to determine the formula once you can answer these questions.
   (a) Are the payments at the BEGINNING or END of each period?
      - If it is at the BEGINNING, then it is an annuity DUE.
      - If it is at the END, then it is an ORDINARY ANNUITY.
        Important Note: If it is a LOAN (car loan, home loan, student loan), then it is an ORDINARY annuity as we discussed in class!

   (b) Is the account balance GROWING (PV = 0, FV = ?) or SHRINKING (PV = ?, FV = 0)?
      - If each payment is a DEPOSIT or INVESTMENT, then the account is GROWING, so it is a FUTURE VALUE question.
      - If each payment is a WITHDRAWAL, then the account balance is shrinking, so it is a PRESENT VALUE question.
        Important Note: If it is a LOAN (car loan, home loan, student loan), then it is a PRESENT VALUE question as we discussed in class!

   Then use the appropriate annuity formula:

	Ordinary (END)	Due (BEGINNING)
Future Value = ??	$F = R\left[\frac{(1+i)^n - 1}{i}\right]$	$F = R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$
Present Value = ??	$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$	$P = R\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$

4. Once you have answered these questions. Label all the variables and solve for any unknowns. Then make sure you answer the original question from the problem!

Okay, now try to set up the problems on the next page.

Compilation of Random Homework and Old Exam Questions:

1. If $4000 is deposited at the end of each quarter in an account that earns 6% compounded quarterly, after how many quarters will the account contain $60,000?
2. If you want to earn 15% annual simple interest on an investment, how much should you pay for a note that will be worth $20,250 in 10 months?
3. How much must you contribute at the beginning of each month in an investment that pays 9%, compounded monthly, if you want the balance to be $130,000 at the end of 18 years?
4. What present value amounts to $290,000 if it is invested at 7%, compounded semiannually, for 11 years?
5. What amount must be set aside now to generate payments of $40,000 at the beginning of each year for the next 11 years if money is worth 5.19%, compounded annually?
6. $5,000 is invested for 4 months at an annual simple interest rate of 12%, how much interest will be earned?
7. John Fare purchased $18,000 worth of equipment by making a $3000 down payment and promising to pay the remainder of the cost in semiannual payments over the next 3 years. The interest rate on the debt is 6%, compounded semiannually. What is the size of the payments for the loan?
8. A year-end bonus of $25,000 will generate how much money at the beginning of each month for the next year, if it can be invested at 6.18%, compounded monthly?
9. A recent graduate's student loans total $17,000. If these loans are at 4.7%, compounded quarterly, for 15 years, what are the quarterly payments?
10. A man buys a car for $39,000. If the interest rate on the loan is 12%, compounded monthly, and if he wants to make monthly payments of $800 for 36 months, how much must he put down?
11. You inherit $400,000 and place all the money in an account earning 1.6%, compounded annually. If you withdraw $10,000 at the end of each year, how long will the money last?
12. A couple needs $20,000 as a down payment for a home. If they invest the $15,000 they have at 8% compounded quarterly, how long will it take for the money to grow into $20,000?
13. How much must you deposit in an account paying 3%, compounded continuously, in order to have $500,000 after 33.5 years?
14. A woman paid $5,000 down for a car and agreed to make payments of $330 at the end of each month, for 36 months. If money is worth 3%, compounded monthly, how much would the car have cost if she had paid cash?
15. What interest will be earned if $6100 is invested for 5 years at 10% compounded monthly?
16. Jack has $70,000 in an account which earns 4%, compounded annually. If he withdraws $5,000 from this account at the beginning of each year, how long until he runs out of money in the account?
17. A man makes $3,000 contributions at the end of each half-year to a retirement account for a period of 8 years. The account earns 4.2%, compounded semiannually. For the next 10 years, he makes no additional contributions and no withdrawals. Find the balance of the account after the 18 years.

How to set up each problem from the previous page:

1. '...deposited at the end of each quarter...' $\Rightarrow$ Ordinary Annuity
   '...after how many quarters WILL the account contain ...' $\Rightarrow$ Future Value
   $F = 60000$, $R = 4000$, $r = 0.06$, $m = 4$, $i = \frac{0.06}{4}$, $n =$ number of quarters $= ??$
   Set up and solve for n. Solution: $n = 13.63$ rounds to 14 quarters.

2. NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
   'Simple Interest'
   $P = ??$, $F = 20250$, $r = 0.15$, $t = \frac{10}{12}$.
   Set up and solve for P. Solution: $P = \$18,000$.

3. '...contribute at the beginning of each month...' $\Rightarrow$ Annuity Due
   '...want the balance to be ... at the end of 18 years.' $\Rightarrow$ Future Value
   $F = 130000$, $R = ??$, $r = 0.09$, $m = 12$, $t = 18$, $i = \frac{0.09}{12}$, $n = 12 \cdot 18$
   Set up and solve for R. Solution: $R = \$240.57$.

4. NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
   '...compounded semi-annually...' $\Rightarrow$ Discrete Compounding
   $P = ??$, $F = 290000$, $r = 0.07$, $m = 2$, $t = 7$
   Set up and solve for P. Solution: $P = \$179,157$.

5. '...payments ... at the beginning ...' $\Rightarrow$ Annuity Due
   '...set aside now to generate payments...' $\Rightarrow$ Present Value
   $P = ??$, $R = 40000$, $r = 0.0519$, $m = 1$, $t = 11$, $i = \frac{0.0519}{1}$, $n = 1 \cdot 11$
   Set up and solve for P. Solution: $P = \$346,039$.

6. NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
   'Simple Interest'
   $P = 5000$, $F = ??$, $r = 0.12$, $t = \frac{4}{12}$.
   Set up and solve for F. Then compute interest $= F - P$. Solution: Interest $= \$200$.

7. It is a LOAN question $\Rightarrow$ Ordinary Annuity, Present Value
   $P = 18000 - 3000 = 15000$, $R = ??$, $r = 0.06$, $m = 2$, $t = 3$, $i = \frac{0.06}{2}$, $n = 2 \cdot 3$. Set up and solve for R. Solution: $R = \$2768.96$.

8. '... at the beginning of each month...' $\Rightarrow$ Annuity Due
   '... \$25000 will generate how much money at the beginning of each month...' $\Rightarrow$ Present Value
   $P = 25000$, $R = ??$, $r = 0.0618$, $m = 12$, $t = 1$, $i = \frac{0.0618}{12}$, $n = 12 \cdot 1$
   Set up and solve for R. Solution: $R = \$2142.69$.

9. It is a LOAN problem $\Rightarrow$ Ordinary Annuity, Present Value
   $P = 17000$, $R = ??$, $r = 0.047$, $m = 4$, $t = 15$, $i = \frac{0.047}{4}$, $n = 4 \cdot 15$
   Set up and solve for R. Solution: $R = \$396.44$.

10. It is a LOAN problem $\Rightarrow$ Ordinary Annuity, Present Value
    $P = ??$ (You don't know the loan amount because you don't yet know the down payment!)
    $R = 800$, $r = 0.12$, $m = 12$, $t = 3$, $i = \frac{0.12}{12}$, $n = 12 \cdot 3 = 36$
    Set up and solve for P. The down payment will be the difference between P and \$39,000.
    Solution: Down Payment = \$14,914.

11. '...at the end of each year...' $\Rightarrow$ Ordinary Annuity
'You inherit \$400,000... you withdraw ...' $\Rightarrow$ Present Value
$P = 400000$, $R = 10000$, $r = 0.016$, $m = 1$, $t = ??$, $i = \frac{0.016}{1}$, $n = 1 \cdot t = t$
Set up and solve for t. Solution: $t = 64.36$ years

12. NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
'...compounded quarterly...' $\Rightarrow$ Discrete Compounding
$P = 15000$, $F = 20000$, $r = 0.08$, $m = 4$, $t = ??$
Set up and solve for t. Solution: $t = 3.63$ years.

13. NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
'...compounded continuously...' $\Rightarrow$ Continuous Compounding
$P = ??$, $F = 500000$, $r = 0.03$, $t = 33.5$
Set up and solve for P. Solution: $P = \$183,022$.

14. It is a LOAN problem $\Rightarrow$ Ordinary Annuity, Present Value
$P = ??$ (You don't know the loan amount all you know is the down payment!)
$R = 330$, $r = 0.03$, $m = 12$, $t = 3$, $i = \frac{0.03}{12}$, $n = 12 \cdot 3 = 36$
Set up and solve for P. The original amount will be \$5000 more than this. So your answer will be $P + 5000$. Solution: \$16,347.50

15. NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
'...compounded monthly...' $\Rightarrow$ Discrete Compounding
$P = 6100$, $F = ??$, $r = 0.10$, $m = 12$, $t = 5$
Set up and solve for F. Then compute interest $= F - P$. Solution: \$3,936.40

16. '...at the beginning of each year...' $\Rightarrow$ Annuity Due
'Jack has \$70,000... he withdraws...' $\Rightarrow$ Present Value
$P = 70000$, $R = 5000$, $r = 0.04$, $m = 1$, $t = ??$, $i = \frac{0.04}{1}$, $n = 1 \cdot t = t$
Set up and solve for t. Solution: $t = 19.71$ years.

17. Two different questions here:

Question 1: "A man makes \$3,000 contributions at the end of each half-year to a retirement account for a period of 8 years. The account earns 4.2%, compounded semiannually."
Answer to Question 1:
'...end of each half-year...' $\Rightarrow$ Ordinary Annuity
'...makes \$3,000 contributions ... to a retirement account...' $\Rightarrow$ Future Value
$F = ??$, $R = 3000$, $r = 0.042$, $m = 2$, $t = 8$, $i = \frac{0.042}{2}$, $n = 2 \cdot 8$.
Set up and compute the value of F. That will be the value of the account in 8 years.
Solution: \$56,354.10

Question 2: "For the next 10 years, he makes no additional contributions and no withdrawals. Find the balance of the account after the 18 years."
Answer to Question 2:
NO mention of regular payments! $\Rightarrow$ Lump Sum Problem.
'...compounded semiannually...' $\Rightarrow$ Discrete Compounding
$P =$ value you got from Question 1!
$F = ??$, $r = 0.042$, $m = 2$, $t = 10$
Set up and solve for F. Solution: \$85,396.60.