6.2 Compound Interest
We say interest is **compounding** if it is computed on the entire balance (principal and previous interest). And we have

\[ F = P(1 + i)^n = P \left(1 + \frac{r}{m}\right)^{mt} \]

Unless otherwise stated, **assume interest is compounded**.

(If you are doing 6.1 homework or if it say “simple interest”, then it is simple interest. **All other times** assume interest is compounding).
Quick Examples:
(a) Harry invests $5000 in an account earning 5% per year. What is the balance in 20 years?

(b) Ron invests $5000 in an account earning 5% annually, compounded quarterly. What is the balance in 20 years?
Continuous Compounding
Assume you invest $5000 and \( r = 0.03 \).

\[
F = 5000 \left( 1 + \frac{0.03}{m} \right)^{mt}
\]

<table>
<thead>
<tr>
<th>m</th>
<th>Balance one yr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$5151.12500</td>
</tr>
<tr>
<td>4</td>
<td>$5151.69595</td>
</tr>
<tr>
<td>12</td>
<td>$5152.07979</td>
</tr>
<tr>
<td>365</td>
<td>$5152.26632</td>
</tr>
<tr>
<td>87605</td>
<td>$5152.27241</td>
</tr>
<tr>
<td>525600</td>
<td>$5152.27267</td>
</tr>
<tr>
<td>3153600</td>
<td>$5152.27267</td>
</tr>
</tbody>
</table>

The value this is approaching is called the value from **continuous compounding**. And it is also given by

\[
F = Pe^{rt}
\]

\[
F = 5000e^{0.03(1)} \approx 5152.27267
\]
Quick Examples:
(a) How much must you invest at 8%, compounded monthly in order to have $10,000 in 5 years?

(b) You invest $500 in an account earning 3%, compounded quarterly. How long until you have $5000?
(c) You invest $75 in an account where interest is computed semi-annually. After 7 years, the balance is $210. What is the nominal rate, \( r \)?

(d) You place 1000 into an account that pays 4%, compounded continuously. How long does it take to double your money?
Summary:

For ALL problems in chapter 6:

1. Identify the type of account.
2. Input given facts.
3. Solve for the unknown.

Algebra Notes:

\[ F = P(1 + i)^n = P \left(1 + \frac{r}{m}\right)^{mt} \]

To solve for \( P \), you just divide.
To solve for \( r \), a root will be needed.
To solve for \( t \), a \( \ln(\ ) \) will be needed.

\[ F = Pe^{rt} \]

To solve for \( P \), you just divide.
To solve for \( r \), a \( \ln(\ ) \) will be needed.
To solve for \( t \), a \( \ln(\ ) \) will be needed.
Question:
Which is best?
A: 4%, compounded semi-annually
B: 3.97%, compounded monthly
C: 3.955%, compounded continuously

Compute the APY for each & compare!

Annual Percentage Yield (APY) is the actual percentage change in one year.

We get this by plugging in $t = 1$ and look at the amount we are multiplying by as a percentage increase:

$$APY = \left(1 + \frac{r}{m}\right)^{m(1)} - 1 \cdot 100\%$$

$$APY = \left[e^{r(1)} - 1\right] \cdot 100\%$$