In class and on this review sheet, I will review some key points of the course so far. However, you are expected to know ALL material that we have covered up to this point.

**The Quick and Dirty Review**

1. **The Prologue**
   - Slopes of lines, solving equations involving lines.
   - Solving quadratic equations and the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

2. **Average (Overall) Rates of Change**
   - ATS (Car), Average (Overall) Output (Reservoir), AR and AC (Shin Polish), CGPA (Grade Point), etc.
   - The slope of the secant line through $x = 0$. If the graph goes through the point $(0, 0)$, then this is the slope of the diagonal line. The functional notation is $f(x) - f(0) \over x - 0$. If $f(0) = 0$, then $f(x) \over x$.
   - Given an average (overall) rate of change, we draw a line through $x = 0$ with the given rate (i.e. slope) and see where it intersects the graph.

3. **Average (Incremental) Rates of Change**
   - As (Car), Average Change each half hour (Reservoir), MR and MC (Shin Polish), QGPA (Grade Point), etc.
   - The slope of the secant line from $x = a$ to $x = b$. The functional notation is $\frac{f(b) - f(a)}{b - a}$.
   - Given an average (incremental) rate of change, we draw a reference through $x = 0$ with the given rate (i.e. slope) and slide our ruler parallel to this slope until we find an appropriate interval.

4. **Change in “Something”**
   - Change in Distance (Car), Change in Output (Reservoir), Change in TR and Change in TC (Shin Polish), Change in Grade Points (Grade Point), etc.
   - The change in height from $x = a$ to $x = b$. The functional notation is $f(b) - f(a)$.
   - Recall, we often fix $\Delta x = \text{the change in } x$, and then we compute values of $\Delta f(x) = \text{change in } f(x)$.

5. **Translation**
   - Be able to translate between the 3 languages: English, Graph, and Functional

6. **Total Revenue (TR) and Total Cost (TC)**
   - Finding the quantity that gives biggest profit by looking at the TR and TC graphs.
   - Finding the break even price (BEP) and shutdown price (SDP) and understanding what these represent.
   - Understand how profit, BEP, and SDP can be explored using MR, AC and AVC.
The More Elaborate Review

1. Prologue
   
   - Slope = \( \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1} \) if \((x_1, y_1)\) and \((x_2, y_2)\) are two points on the line. (Problems 1 - 6)
   - Solving when lines intersect. (Problem 7)
   - Quadratic equations, vertex formula, and the intersection of quadratics and lines. (Problems 8 - 11)

2. Worksheet 1 - The Car Story
   
   - Average Trip Speed = \( \overline{\text{ATS}} = \frac{\text{Total Change in Distance (so far)}}{\text{Total Change in Time (so far)}} \) and Average Speed = \( \overline{\text{AS}} = \frac{\text{Change in Distance}}{\text{Change in Time}} \) (Problems 1 - 8)
   - Note that we fixed \( \Delta t = 5 \) and computed \( \Delta D \). The labeled \( C \) is the distance covered in a five minute interval.
   - \( \overline{\text{ATS}} \) = slope of a secant through the \( t = 0 \), \( \overline{\text{AS}} \) = slope of a secant through \( t = a \) and \( t = b \). Note: We can use this to find the times that give lowest or highest \( \overline{\text{ATS}} \) and \( \overline{\text{AS}} \). (Problems 11-12, 1-I and 1-II)

3. Worksheet 2 - The Reservoir Story
   
   - \( U(t) \) = usage over the last half hour, we computed \( O(t) \) = total output \( t \) hours after noon, \( I(t) \) = total input \( t \) hours after noon. (Problems 1 - 4)
   - Whenever input below output on the graph, there is a shortage. If we want to find the least amount of water that has to be in the reservoir at noon we find the greatest distance that output is above input. (Problems 5 - 8)

4. Worksheet 3 - The Print Shop Story
   
   - \( TR \) = total revenue, \( TC \) = total cost, \( MR \) = marginal revenue = change in \( TR \) when quantity \( q \) is increased by 1, \( MC \) = marginal cost. (Problems 1 - 7)
   - \( P \) = profit = \( TR - TC \), the quantity when \( TR \) has the greatest vertical distance above \( TC \) is where profit is maximized. (Problems 8 - 11)
   - Marginal Analysis: If \( MR > MC \), then profit increases when we sell one more unit. If \( MR < MC \), then profit decreases when we sell one more unit. Profit is maximized at the first quantity at which \( MR \) falls below \( MC \). (Problems 12 - 14)

5. Worksheet 4 - Increments
   
   - We discussed the similarities between the three stories and how there are total amounts \( (D, O, \text{ and } TR) \) and incremental amounts \( (C, U, \text{ and } MR) \). (Problems 1 - 8, 4-I - 4-III)

6. Worksheet 5 - Increments and Speeds
   
   - We noted how \( \overline{\text{ATS}} \) in the car story corresponds to \( AR = \text{average revenue} = \text{price} \) in the print shop story. We also noted how \( \overline{\text{AS}} \) corresponds to \( MR \). (Problems 1 - 7)
   - The grade point story was introduced a another example. (Problems 8 - 15, 5-I)
7. Worksheet 6 - The Lagging Car

- Functional notation was introduced in this worksheet. We began the basic of translating into functional notation. (Problems 1-13, 6-I)

8. Worksheet 7 - A Reservoir in Three Languages

- Translating between English, Graph, and Functional languages. (Problems 1 - 7, 7-I)

9. Worksheet 8 - Increments and Reference Lines

- This is an important section! We can answer 3 major questions by translating to graphical language. (Problems 1, 2, 8-I - 8-III)
  
  (a) Average (Overall) Rate of Change - Draw a line with the given slope through $x = 0$ and see where it intersects the graph. (Problem 3)
  
  (b) Average Rate of Change over an Interval - Draw a reference line with the given slope through $x = 0$ and slide the ruler parallel to the line until you find an interval of the appropriate length. (Problems 4, 5, 7)
  
  (c) Change in “Something” - Divide both sides by the length the the interval to get a slope and proceed as in (b). (Problem 6)

10. Worksheet 9 - Analysis of Cost I

- $FC = $ fixed cost (rent, etc.), $VC = $ variable cost, $TC = FC + VC$.

- Given a market price $p$, we can draw a straight line graph for $TR$. The breakeven price ($BEP$) is the slope of the lowest line through $(0, 0)$ that crosses the $TC$ graph. The shutdown price ($SDP$) is the slope of the lowest line through $(0, 0)$ that crosses the $VC$ graph. (Problems 1 - 14, 9-I)
  - If $p > BEP$, then some quantities make a profit.
  - If $p < BEP$, then no quantities make a profit
  - If $p > SDP$, then some quantities will make more money than $FC$. (DON’T SHUT DOWN)
  - If $p < SDP$, then no quantities will make more money than $FC$. (SHUT DOWN)

11. Worksheet 10 - Analysis of Cost II

- $AVC(q) = \frac{VC(q)}{q}$ is the slope of the diagonal line through the graph of $VC$. (Problems 1 - 12, 10-I and 10-II)

- $BEP =$ the intersection of $MC$ and $AC$.

- $SDP =$ the intersection of $MC$ and $AVC$. 