Math 111 A - Autumn 2005
Exam 1
October 20, 2005

Name: 

Instructor Key

Section: 

Version 1

Student ID Number: 

TA's Name: 

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- You are allowed to use a calculator, a ruler, and one hand-written 8.5 by 11 inch page of notes.

- Check that your exam contains all the problems listed above.

- You must show your work on all problems. The correct answer with no supporting work may result in no credit. On problems in which you use a graph, draw lines clearly on the graph.

- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.

- Raise your hand if you have a question.

- Put your name on your sheet of notes and turn it in with the exam.

- Any student found engaging in academic misconduct will receive a score of 0 on this exam.

- You have 50 minutes to complete the exam.

GOOD LUCK!
1. (16 points) The following is the graph of the distance traveled by a car during the first hour of a trip. We abbreviate the distance at time, $t$, by $D(t)$.

![Graph of distance over time](image_url)

(a) Compute the car's average speed from $t = 20$ to $t = 40$.

FIND THE SLOPE OF THE SECANT LINE THROUGH THE DISTANCE GRAPH AT $t = 20$ AND $t = 40$.

\[
\frac{45 - 35}{40 - 15} = 0.4
\]

ANSWER: average speed $= \frac{1}{2.6}$ mpm

(b) What is the car's lowest average trip speed?

FIND THE SLOPE OF THE LOWEST DIAGONAL LINE THROUGH THE DISTANCE GRAPH.

\[
\delta_0 = 0.75
\]

ANSWER: $\frac{1}{1.75}$ mpm

(c) Find a time $t$ where the average trip speed is 1.0 mpm.

FIND WHERE THE DIAGONAL LINE WITH SLOPE 1 INTERSECTS THE DIST. GRAPH.

about 45

ANSWER: $t = \frac{43.47}{25}$ min

(d) Find two values for $t$ such that

\[
D(t + 5) - D(t) = \frac{1}{2}
\]

Note: This is the range of acceptable answers for $t$.

ANSWER: $t = \frac{21.29}{25}$ min and $\frac{56.64}{60}$ min

DRAW A REFERENCE LINE WITH SLOPE \( \frac{1}{2} \) AND FIND THE BEGINNINGS OF 2 5-MINUTE INTERVALS WITH THIS SLOPE.
2. (20 points) You own a clock shop. The following is the graph of the total cost $TC(q)$ and variable cost $VC(q)$ when selling $q$ clocks.

(a) i. Draw a graph of a straight line representing the total revenue ($TR$) if the market price is $3.00 per clock. Draw a diagonal line with slope = 3.
ii. If the market price is $3.00 per clock, determine the quantity where profit is maximum. Find greatest vertical distance ($TR > TC$) or find $q$ s.t. slope of $TC = $slope of $TR$.

ANSWER: $q = \frac{124}{60} = 2.06$

(b) Find the break even price ($BEP$).

Find the slope of the lowest diagonal line through $TC$.

ANSWER: price per clock = \[
\begin{bmatrix}
45
2.25
\end{bmatrix}
\] dollars

(c) Find the shutdown price ($SDP$).

Find the slope of the lowest diagonal line through $VC$.

ANSWER: price per clock = \[
\begin{bmatrix}
0.45
0.7
\end{bmatrix}
\] dollars

(d) Find a 10 minute interval where $\frac{VC(q)}{q}$ decreases and then increases.

Find where the slope of the diagonal line through $VC$ decreases, then increases.

ANSWER: interval starts at $q = \frac{51 - 59}{6}$ and ends at $q = \frac{61 - 69}{6}$

SHOULD INCLUDE SDP LOCATION
3. (6 points) Let $H(t)$ be the height (in feet) of an evergreen tree $t$ years after being planted.

(a) Translate the following into functional notation: "The tree grows 10 feet during the $h$-minute interval starting at 6 years after being planted."

$$H(6+h) - H(6) = 10$$

(b) Translate the following into English: \( \frac{H(9) - H(4)}{5} > 3 \)

The rate of change of the height of the tree from $t = 4$ to $t = 9$ yrs is bigger than \( \frac{3}{1} \) ft per year.

4. (8 points)

(a) Solve for $x$ and $y$:

\[
\begin{align*}
2x + 5 &= 3y \\
x - 2 &= y
\end{align*}
\]

\[
\begin{align*}
2x + 5 &= 3(x - 2) \\
2x + 5 &= 3x - 6 \\
11 &= x \\
y &= 9
\end{align*}
\]

(b) Find all solutions for $q$: \( q^2 - 29q + 60 = -40 \)

\[
q^2 - 29q + 100 = 0 + 2
\]

\[
(q - 25)(q - 4) = 0
\]

\[
q = 4 \quad q = 25
\]