A Square Ellipse?

Here is the problem from an old exam which we are visualizing in this print…

6. (10 total points) The goal of this problem is to compute the volume of the container pictured below on the left. The top rim of the container is an ellipse with the dimensions indicated in the picture on the right. Cross-sections of the container perpendicular to the long axis of the elliptical rim are squares (a representative cross-section is shown).

Top View

2b meters

2a meters
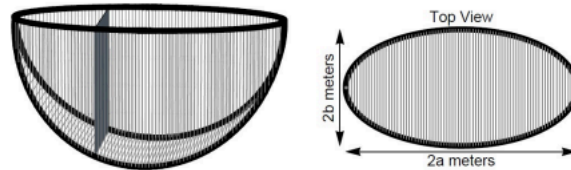
Recall that the equation of an ellipse centered at the origin, symmetric with respect to both the x- and y-axes, and with x-intercepts $x = \pm a$ and y-intercepts $y = \pm b$, is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$
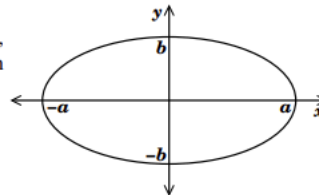
(a) (5 points) Express the volume of the container as a definite integral.

Go ahead and give it a shot. Then ask your own volume questions.