1. In spherical coordinates the sphere is \( \rho = \sqrt{2} \) and the cone is \( \rho \cos \phi = \rho \sin \phi \), i.e., \( \cos \phi = \sin \phi \), i.e., \( \phi = \frac{1}{4} \pi \). Also \( x = \rho \sin \phi \cos \theta \), \( z = \rho \cos \phi \), and \( dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \), so
\[
\iiint_E e^{xz} \, dV = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{2}} e^{\rho^2 \cos \phi \sin \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta.
\]

2. The iterated integral represents the double integral over the region \( D \) between the line \( y = 2x \) and the parabola \( y = 3 - x^2 \), which meet at \((-3, -6)\) and \((1, 2)\). So as an iterated integral in the opposite order, it is
\[
\int_{-6}^{2} \int_{-\sqrt{3-y}}^{\sqrt{3-y}} f(x, y) \, dx \, dy + \int_{2}^{3} \int_{-\sqrt{3-y}}^{\sqrt{3-y}} f(x, y) \, dx \, dy.
\]
(This should be clear if you draw a sketch of \( D \).)

3. The sides of the triangle are the lines \( y = 0 \), \( y = -x \), and \( y = x + 2 \). The inverse transformation of \( u = x+y \), \( v = x-y \) is \( x = \frac{1}{2}(u+v) \), \( y = \frac{1}{2}(u-v) \), so the three lines just described correspond to the lines \( u = v \), \( u = 0 \), and \( v = -2 \), and the image of \( D \) is the triangle with vertices \((0, 0)\), \((0, -2)\), and \((-2, -2)\). Also, the Jacobian is
\[
\frac{\partial (x, y)}{\partial (u, v)} = \det \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = -\frac{1}{2},
\]
and its absolute value (which is what you need) is \( \frac{1}{2} \). Thus
\[
\iint_D \cos \frac{\pi(x+y)}{2(x-y)} \, dA = \int_{-2}^{0} \int_{-\frac{u}{2}}^{\frac{u}{2}} \cos \frac{\pi u}{2v} \frac{1}{2} \, du \, dv = \int_{-2}^{0} \frac{v}{\pi} \sin \frac{\pi u}{2v} \left. \right|_{u=0}^{0} \, dv = -\frac{1}{\pi} \int_{-2}^{0} v \, dv = \frac{2}{\pi}.
\]

4. We have \( \text{div} \, \mathbf{F} = 2x + 0 + x = 3x \), so by the divergence theorem,
\[
\iiint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E 3x \, dV = \int_0^{6} \int_0^{(6-x)/2} \int_0^{(6-x-2y)/3} 3x \, dz \, dy \, dx
\]
\[
= \int_0^{6} \int_0^{(6-x)/2} (6-x-2y) \, x \, dy \, dx = \int_0^{6} \left[ x(6-x) y - xy^2 \right]_{y=0}^{(6-x)/2} \, dx
\]
\[
= \int_0^{6} \frac{1}{4} x(6-x)^2 \, dx = \left[ \frac{9}{2} x^2 - x^3 + \frac{1}{16} x^4 \right]_0^6 = 162 - 216 + 81 = 27.
\]

5. (a) \( \text{curl} \, \mathbf{F} = 0 \) and \( \text{div} \, \mathbf{F} = \frac{1}{2} e^{x/2} - 25x \cos(3y + 4z) - 2z \).
(b) Yes, \( f(x, y, z) = 2e^{x/2} + x \sin(3y + 4z) - \frac{1}{3} z^3 + c \).
(c) No, because \( \text{div} \, \mathbf{F} \neq 0 \).
6. For the integral over $C_1$, use Green’s theorem. (The orientation is “wrong,” so there’s an extra minus sign.) Denoting the region inside the ellipse by $D$,

$$
\int_{C_1} (xy \, dx - x^2 \, dy) = -\iint_D \left[ \frac{\partial(-x^2)}{\partial x} - \frac{\partial(xy)}{\partial y} \right] \, dA = \iint_D 3x \, dA = 0
$$

because $3x$ is an odd function and $D$ is symmetric about the line $x = 0$.

There are a couple of ways to do the integral over $C_2$. You can take $y$ as the parameter (running backwards from $3$ to $-3$); then $x = \frac{2}{3}\sqrt{9 - y^2}$ and $dx = -\frac{2}{3}(y/\sqrt{9 - y^2}) \, dy$, so

$$
\int_C (xy \, dx - x^2 \, dy) = \int_{-3}^3 \frac{4}{9}[ - y^2 - (9 - y^2)] \, dy = \frac{4}{9}(-9)(-6) = 24.
$$

Or, you can you can use trig functions to parametrize, say $x = 2 \cos t$, $y = 3 \cos t$, $0 \leq t \leq \pi$ (other variations are possible). Then $dx = 2 \sin t \, dt$ and $dy = -3 \sin t \, dt$, so

$$
\int_C (xy \, dx - x^2 \, dy) = \int_0^\pi (12 \cos^2 t \sin t + 12 \sin^3 t) \, dt = \int_0^\pi 12 \sin t \, dt = -12 \cos t|_0^\pi = 24.
$$

For the scalar line integral, these two parametrizations give

$$
\int_C x \, ds = \int_{-3}^3 2\sqrt{9 - y^2}\sqrt{4y^2/9(9 - y^2)} + 1 \, dy = \int_0^\pi 3 \sin t \sqrt{4 \cos^2 t + 9 \sin^2 t} \, dt.
$$

(Yes, the second integral is $\int_{-3}^3$. We have $ds = \sqrt{dx^2 + dy^2}$, which equals $\sqrt{(dx/dy)^2 + 1} \, dy$ only if the increment $dy$ is positive, i.e., $y$ goes from smaller to larger. Otherwise there’s a minus sign that compensates for reversing the limits of integration.)

7. The surface is parametrized by $\mathbf{r}(\theta, z) = \sqrt{1 + z^2}(\cos \theta) \mathbf{i} + \sqrt{1 + z^2}(\sin \theta) \mathbf{j} + z \mathbf{k}$ ($0 \leq \theta \leq 2\pi$, $0 \leq z \leq 1$), so one calculates that $\mathbf{r}_\theta \times \mathbf{r}_z = \sqrt{1 + z^2}(\cos \theta) \mathbf{i} + \sqrt{1 + z^2}(\sin \theta) \mathbf{j} - z \mathbf{k} = xi + yj - zk$ (with the right orientation: the horizontal part $xi + yj$ points outward). Thus $\mathbf{F} \cdot (\mathbf{r}_\theta \times \mathbf{r}_z) = x^2 + y^2 - z^2$, which equals 1 on the surface $S$, so

$$
\iint_S \mathbf{F} \cdot d\mathbf{S} = \int_0^1 \int_0^{2\pi} 1 \, d\theta \, dz = 2\pi.
$$

Also $dS = |\mathbf{r}_\theta \times \mathbf{r}_z| \, d\theta \, dz = \sqrt{x^2 + y^2 + z^2} \, d\theta \, dz = \sqrt{1 + 2z^2} \, d\theta \, dz$, so

$$
\iint_S z \, dS = \int_0^1 \int_0^{2\pi} z\sqrt{1 + 2z^2} \, d\theta \, dz = 2\pi \cdot 1 \cdot \frac{2}{3}(1 + 2z^2)^{3/2}|_0^1 = \frac{\pi}{3}(3^{3/2} - 1).
$$

8. Use Stokes: a bit of calculation shows that $\text{curl} \, \mathbf{F} = i + 2j + 3k$ and $\mathbf{r}_u \times \mathbf{r}_v = -6ui + 2uj + 2k$ (the correct orientation), so

$$
\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl} \, \mathbf{F} \cdot d\mathbf{S} = \int_0^1 \int_0^1 (-2u + 6) \, du \, dv = 5.
$$