1. Consider two vector fields: \( \mathbf{F} = (x + z, 1, x) \) and \( \mathbf{G} = (y, -x, e^z) \).

a) For each of the two fields, determine whether it is conservative. Show your reasoning! Give a potential function for each conservative field.

b) Let \( C \) be the curve from \((0,0,0)\) to \((4,20)\) along the intersection of the surfaces defined by \( x^2 + y^2 = z \) and \( x = 2y \). Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) and \( \int_C \mathbf{G} \cdot d\mathbf{r} \).

2. The function \( g \) of three variables is given by \( g(x,y,z) = xz^2 + y - e^{6} \).

   a) Suppose \( \mathbf{r}(t) \) is a parametrized curve; we do not know the formulas for \( \mathbf{r}(t) \), but we know that \( \mathbf{r}(5) = (2,-7,3) \) and \( \mathbf{r}'(5) = (-1,\pi,2) \). Define a new function \( h(t) = g(\mathbf{r}(t)) \); find \( h'(5) \).

   b) Find the equation of the tangent plane to the level set for \( g \) through the point \((2,-7,3)\).

   c) Suppose you are at the point \((2,-7,3)\), and you want to start moving in a direction so that \( g \) stays constant. Give one possible direction for which this is true.

3. Let \( S \) be the part of the surface \( y = z^2 \) inside the cylinder \( x^2 + z^2 = 4 \), oriented by the normal with positive \( j \) component.

   a) Give a parametrization \( \mathbf{r}(u,v) \) of \( S \), including specifying the domain (that is, the bounds on \( u,v \)). Does \( \mathbf{r}_u \times \mathbf{r}_v \) give the orientation specified, or the opposite orientation?

   b) Give a parametrization of the boundary curve \( C \) of \( S \) as a function of \( t \), including specifying the interval for \( t \). Does your parametrization give the orientation of \( C \) consistent with the given orientation of \( S \), or the opposite orientation?

   c) Compute \( \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F} = zi + (4 - x^2 - z^2)j - xk \). (You may compute it directly, or use one of the theorems of chapter 16.)

4. Let \( S \) be part of the cylinder \( x^2 + y^2 = 9 \) where \( 0 \leq z \leq 5 \). Let \( f(x,y,z) = 2z \), and let \( \mathbf{F} = \mathbf{i} + \mathbf{k} \).

Determine whether each of the following expressions makes sense. If it doesn’t make sense, say briefly why. If it does make sense, compute it. (Hint: you may be able to reason directly from the meaning of the surface integrals and compute them without setting up a parametrization.)

   a) \( \iint_S f dS \)

   b) \( \iint_S f \cdot dS \)

   c) \( \iint_S \mathbf{F} \cdot d\mathbf{S} \)

5. Reasoning from pictures of vector fields: p. 1044-1045, #17, 18, 47; p. 1054, #23-24; p. 1068, #9-11 (can use ideas from later sections, pp. 1096 and 1103); p. 1104, #19-20.