ADDED PROBLEM 4 ON TUESDAY 11/15.

See also: recent quizzes, the actual quiz and the samples, esp. quiz 3 (on parametrizing surfaces); MT 2 review assignment on WebAssign; and two more, if you need more: p. 1107, #5 & 13 (answers in back of book).

I’ll try to post some final answers or solutions for these 4 problems no later than noon on Thursday.

1. Let \( f(x, y, z) = x \cos(\pi y) + ye^z \).

   (a) Compute \( \nabla f \) at \((2, 3, 1)\).

   (b) A curve \( r(t) \) passes through \((2, 3, 1)\) at \( t = 0 \), so \( r(0) = (2, 3, 1) \). The velocity vector \( r'(0) \) points from \((2, 3, 1)\) towards \((5, 3, 5)\), and the speed there is \( |r'(0)| = 2 \). Find \( r'(0) \) and use it to compute \( \frac{d}{dt} f(r(t)) \) at \( t = 0 \). (Hint if you are stuck: use the chain rule.)

2. Let \( C \) be the curve consisting of the line segments from \((0, 0, 0)\) to \((1, 1, 1)\) and from \((1, 1, 1)\) to \((1, 0, 1)\). Compute the mass of a thin wire bent in the shape of the curve \( C \) if the density at any point is equal to \( \rho(x, y, z) = 2 - z \).

3. Let \( \mathbf{F}(x, y) = (x^3 - 2xy^3)i - 3x^2y^2j \).

   (a) Show that \( \mathbf{F} \) is conservative.

   (b) Find a potential function for \( \mathbf{F} \).

   (c) Evaluate the line integral of \( \mathbf{F} \) along the curve, \( x = \cos^3 t, \ y = \sin^3 t, \ 0 \leq t \leq \pi/2 \).

4. Let \( C \) be the curve of intersection of the plane \( y + z = 5 \) and the cylinder \( x^2 + y^2 = 9 \), going counterclockwise as viewed from above.

   (a) Find a parametrization of \( C \). (Note that you are parametrizing a curve, so your answer should be a function on just one parameter. If that parameter is \( t \), your answer would be in the form \( r(t) = (x(t), y(t), z(t)) \), or just the trio of functions \( x(t), y(t), z(t) \).)

   (b) Use your parametrization to compute \( \int_C \mathbf{F} \cdot d\mathbf{r} \), if \( \mathbf{F} = (x, 2y, -4) \).