12.1 and 12.2 Overview

- **Distance**: \( \text{DISTANCE} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \).

- **Sphere Equation**: \( (x - h)^2 + (y - k)^2 + (z - l)^2 = r^2 \).

- **Horizontal and Vertical Planes**:
  \( x = a \Leftrightarrow \) vertical plane parallel to the \( yz \)-plane at \( x = a \).
  \( y = b \Leftrightarrow \) vertical plane parallel to the \( xz \)-plane at \( y = b \).
  \( z = c \Leftrightarrow \) horizontal plane parallel to the \( xy \)-plane at \( z = c \).

- The magnitude of \( \mathbf{v} = \langle v_1, v_2 \rangle \) is \( |\mathbf{v}| = \sqrt{v_1^2 + v_2^2} \).

- **Scalar multiplication**: If \( c \) is a scalar and \( \mathbf{v} \) is a vector, then \( c\mathbf{v} \) means multiply each component of \( \mathbf{v} \) by \( c \). This scales the magnitude of \( \mathbf{v} \) by a factor \( c \).

- **Equality**: Two vectors are **equal** if they have exactly the same components.

- A **unit vector** is a vector with magnitude one. To get a unit vector in the same direction as \( \mathbf{v} \), you must divide by the length. That is,
  \[
  \frac{1}{|\mathbf{v}|} \mathbf{v} = \text{a unit vector in the same direction as } \mathbf{v}.
  \]
• Vector Addition: If $\mathbf{u}$ and $\mathbf{v}$ are two vectors, then $\mathbf{u} + \mathbf{v}$ is the vector obtained by adding the corresponding components of each vector. Graphically, the sum $\mathbf{u} + \mathbf{v}$ is the diagonal of the parallelogram with sides $\mathbf{u}$ and $\mathbf{v}$. Physically, it can be thought of as the resultant force of the two forces $\mathbf{u}$ and $\mathbf{v}$.

• Representations:
  Bracket notation: $\langle 2, 3, -5 \rangle$
  Standard basis notation: $2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$
  They both are essentially the same and it is a matter of taste. For example, $\langle 2, 3, -5 \rangle = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$.
  The standard basis vectors are simply $\mathbf{i} = \langle 1, 0, 0 \rangle$, $\mathbf{j} = \langle 0, 1, 0 \rangle$, and $\mathbf{k} = \langle 0, 0, 1 \rangle$.

• The angle that $\mathbf{v} = \langle v_1, v_2 \rangle$ makes with the positive $x$-axis can be determined by drawing the vector, making a triangle, labeling the angle and using $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$ or some other appropriate trig function.