Ch. 12 Vector Ops/Lines/Planes

- \( \langle a, b \rangle \ast \langle c, d \rangle = ac + bd \)
- \( \langle a, b, c \rangle \ast \langle d, e, f \rangle = ad + be + cf \)
- \( \langle a, b, c \rangle \times \langle d, e, f \rangle = \langle bf - ce, cd - af, ae - bd \rangle \)
  
  Remember the computational method discussed in class.

Lines

If \( \vec{v} = \langle a, b, c \rangle \) is a vector parallel to the line (direction vector), \((x_0, y_0, z_0)\) is a point on the line, then the equation for points \((x, y, z)\) on the line are given by:

1. Vector form
   \[ \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle \]
2. Parametric form
   \[ \begin{align*}
   x &= x_0 + at \\
   y &= y_0 + bt \\
   z &= z_0 + ct
   \end{align*} \]
3. Symmetric form
   \[ \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \]

Aside

Tangent line at \((x_0, y_0, z_0)\) for \( \vec{r}(t) = \langle f(t), g(t), h(t) \rangle \)

Take \( \vec{v} = \vec{r}'(t) \)
\[ \vec{v} = \langle f'(t), g'(t), h'(t) \rangle \]
**Planes**

If \( \vec{n} = \langle a, b, c \rangle \) is a normal vector to the plane

= orthogonal to any vector parallel to the plane,

\((x_0, y_0, z_0)\) = any point on the plane

then the equation for points \((x, y, z)\) on the plane are given by

| Vector form | \( \langle a, b, c \rangle \cdot (\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) = 0 \) |
| Scalar form | \( a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \) |

**Aside**

**Normal Plane at a point** \((x_0, y_0, z_0)\)

- Take \( \vec{n} = \vec{p}'(t) \)

**Tangent Plane at a point** \((x_0, y_0, z_0)\)

for a surface \( z = f(x, y) \)

- Take \( \vec{n} = \langle 1, -f_x(x_0, y_0), -f_y(x_0, y_0) \rangle \)
Ch. 10  Parametric And Polar

* Know how to work with parametric curves:

\[ x = f(t) \quad y = g(t) \]

which can be written as \( \langle f(t), g(t) \rangle = r(t) \)

* As with curves in 3D,

\[ \vec{v}(t) = \langle f'(t), g'(t) \rangle = \text{velocity vector} \]

\[ \vec{a}(t) = \langle f''(t), g''(t) \rangle = \text{acceleration vector} \]

slope of the tangent line \( \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \)

* Understand Polar Curves

\[ x = r \cos(\theta) \quad r^2 = x^2 + y^2 \]

\[ y = r \sin(\theta) \quad \tan(\theta) = \frac{y}{x} \]

Be able to go back and forth between polar and Cartesian coordinates.

slope of the tangent line \( \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta} \sin(\theta) + r \cos(\theta)}{\frac{dr}{d\theta} \cos(\theta) - r \sin(\theta)} \)

Recall: The equation of a tangent line is of the form

\[ y = mx + b \]

slope of tangent line \( \frac{dy}{dx} \)