Cyclic Sieving for Staircase Plane Partitions via Crystals and Electrical Networks

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University of Waterloo

Joint work with Sam Hopkins and Jesse Kim

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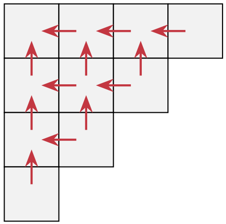
P-partitions and order polynomials

For a finite poset P, a height m *P-partition* is an order-preserving map

$$\pi : P \longrightarrow \{0, 1, \ldots, m\}.$$



δ_4

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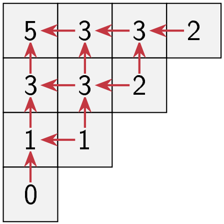

$\pi \in \mathcal{PP}^5(\delta_4)$

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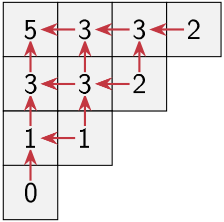

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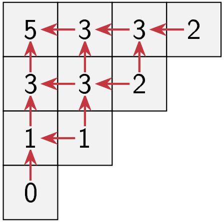

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Piecewise-linear rowmotion, introduced by Einstein–Propp, is an action on the order polytope of a finite poset P. It is a product of *toggles*:

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1	1		
0			

toggle the highlighted 3

The toggle at (i, j) changes only that entry:

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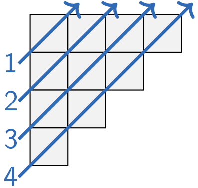

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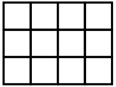
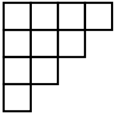


rectangle

[Rhoades, 2010]

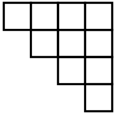
all heights



staircase

[Armstrong–Stump–Thomas, 2013]

height 1



shifted staircase

[Rush–Shi, 2013]

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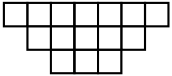

trapezoid

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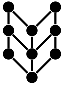

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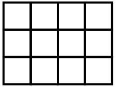

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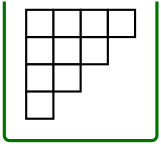
height 2 now



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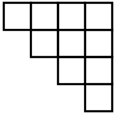

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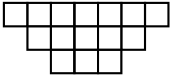

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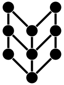

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Good dynamics: cyclic sieving

Let $C = \langle c \rangle$ be a cyclic group of order N acting on a finite set X, and let $f(q) \in \mathbb{N}[q]$.

Definition (Reiner–Stanton–White, 2004)

The triple (X, C, f) exhibits the *cyclic sieving phenomenon* if, for every k,

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Conjecture for staircases

Let

$$\Omega_{\delta_n}(m; q) = \prod_{1 \le i \le j \le n} \frac{1 - q^{i+j+2m}}{1 - q^{i+j}}.$$

For all $m, n \ge 1$,

$$(\mathcal{PP}^m(\delta_n), \langle \mathrm{Row} \rangle, \Omega_{\delta_n}(m; q)) \text{ exhibits CSP}, \qquad |\langle \mathrm{Row} \rangle| = 2(n+1).$$

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Theorem (Hopkins–Kim–P, 26+)

The staircase conjecture is true for $m = 2$.

Proof at a glance

Rhoades-style blueprint: Realize staircase plane partitions as a basis of a representation on which rowmotion acts cyclically, so fixed-point counts become character evaluations.

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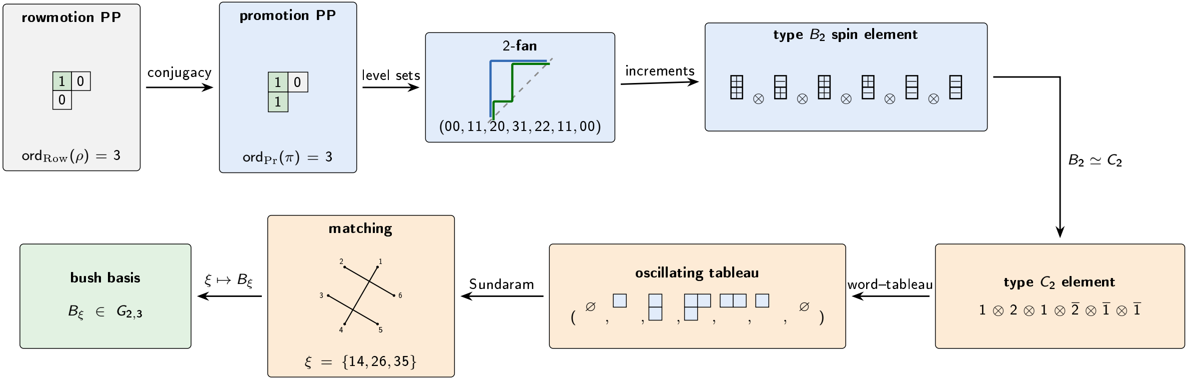
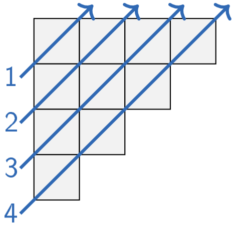


Promotion and Rowmotion

rowmotion



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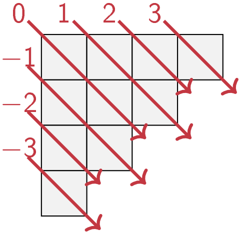

$$\mathrm{Pr} = F_3 \circ \cdots \circ F_{-3}$$

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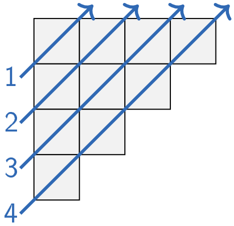

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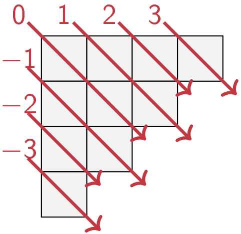

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Theorem (Striker–Williams, 2012)

Pr *and* Row *are conjugate, hence have the same orbit structure.*

Fans of Dyck paths

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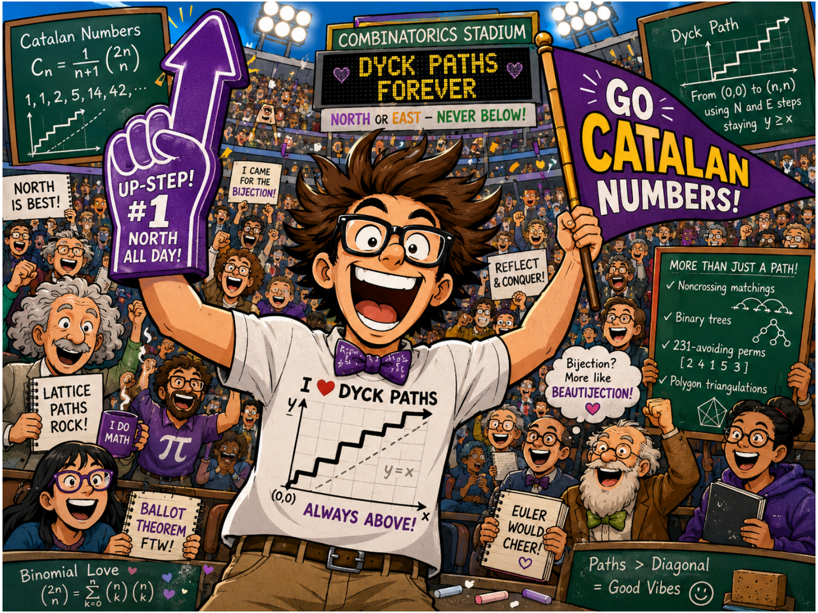

Staircase plane partitions and fans of Dyck paths

3	3	3	3	3	1
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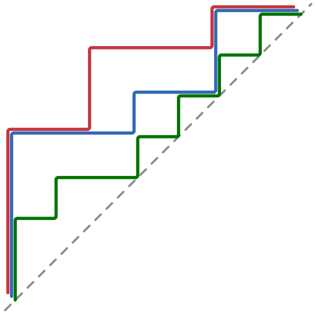
1	1	1			
1	0				
0					

$\pi \in \mathcal{PP}^3(\delta_6)$

$\longleftrightarrow$



$$\mathcal{F} = (000, 111, 222, 331, 442, 331, 220, 311, 420, 311, 200, 111, 220, 111, 000).$$

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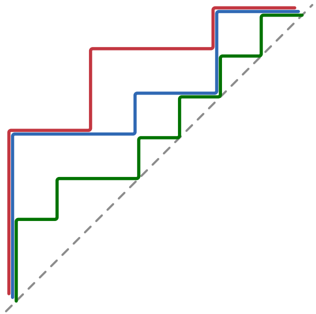

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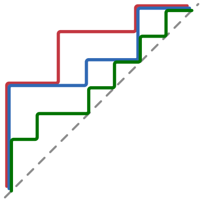
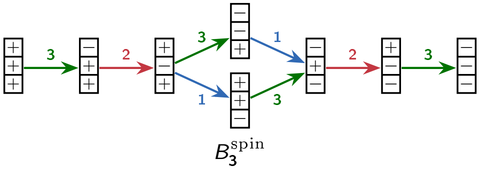
$$\mathcal{PP}^m(\delta_{n-1}) \xrightarrow{\sim} \mathcal{D}_n^m, \qquad \mathrm{Pr} \longleftrightarrow \mathrm{Pr}.$$

The general spin-crystal model





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Each increment $\mu^i - \mu^{i-1} \in \{\pm 1\}^r$ is a vertex of the type B_r spin crystal.

$$b(\mathcal{F}) = \begin{smallmatrix}+\\+\\+\end{smallmatrix} \otimes \begin{smallmatrix}+\\+\\+\end{smallmatrix} \otimes \begin{smallmatrix}-\\+\\+\end{smallmatrix} \otimes \begin{smallmatrix}+\\+\\+\end{smallmatrix} \otimes \begin{smallmatrix}-\\-\\-\end{smallmatrix} \otimes \begin{smallmatrix}-\\-\\-\end{smallmatrix} \otimes \begin{smallmatrix}+\\-\\+\end{smallmatrix} \\ \otimes \begin{smallmatrix}-\\+\\+\end{smallmatrix} \otimes \begin{smallmatrix}+\\-\\-\end{smallmatrix} \otimes \begin{smallmatrix}-\\-\\-\end{smallmatrix} \otimes \begin{smallmatrix}+\\+\\-\end{smallmatrix} \otimes \begin{smallmatrix}-\\+\\+\end{smallmatrix} \otimes \begin{smallmatrix}+\\-\\-\end{smallmatrix} \otimes \begin{smallmatrix}-\\-\\-\end{smallmatrix}.$$

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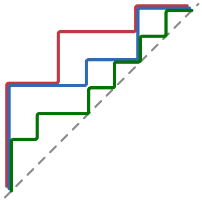

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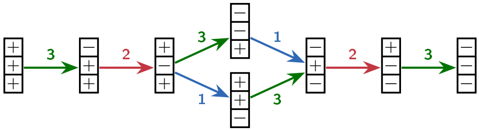


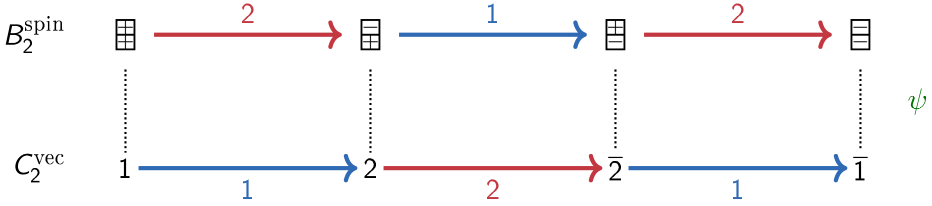
B_3^{spin}

Observation (Pappe–P–Schilling–Simone '24)

$$\mathcal{D}_n^r \xrightarrow{\sim} \mathrm{HW}_0\left(\left(B_r^{\mathrm{spin}}\right)^{\otimes 2n}\right),$$

and fan promotion is the crystal commutor.

The exceptional isomorphism $B_2 \simeq C_2$



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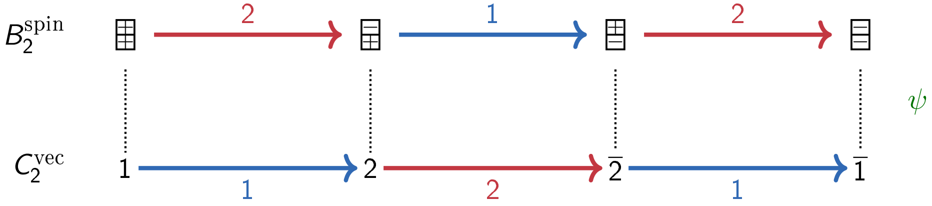

$$\psi^{\otimes 2n} : \mathrm{HW}_0\left((B_2^{\mathrm{spin}})^{\otimes 2n}\right) \xrightarrow{\sim} \mathrm{HW}_0\left((C_2^{\mathrm{vec}})^{\otimes 2n}\right).$$

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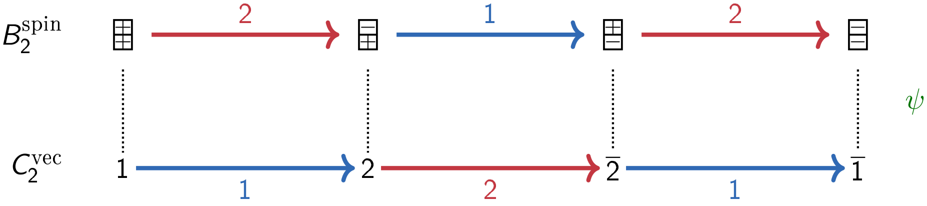

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$b_B =$  $\mapsto 1 \otimes 2 \otimes 1 \otimes \bar{2} \otimes \bar{1} \otimes \bar{1}.$

From a type C word to an oscillating tableau

A weight-zero highest weight element of $(C_2^{\mathrm{vec}})^{\otimes 6}$ is

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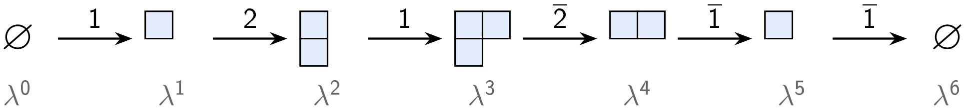

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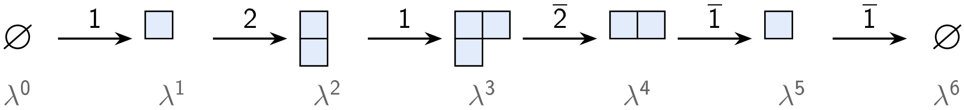


This is a *2-symplectic oscillating tableau*: consecutive shapes differ by one box, every shape has at most two rows, and the sequence begins and ends at $\varnothing$.

Sundaram's map

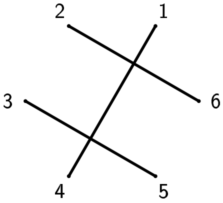
[Sundaram, 1986] gives a bijection

$$\mathcal{M} : \mathcal{O}_r(2n) \xrightarrow{\sim} \mathcal{TC}_{r+1}(2n).$$

between

- weight-zero r-symplectic oscillating tableaux,
- and $(r+1)$-noncrossing perfect matchings

$$(\emptyset, 1, 11, 21, 2, 1, \emptyset) \mapsto$$



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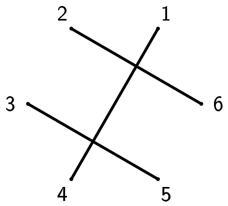
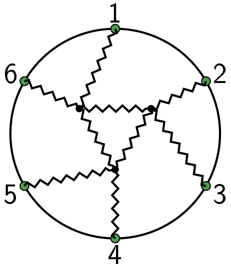


Theorem (P–Rubey–Westbury 2020)

Sundaram's map intertwines crystal promotion and rotation.

Electrical networks and the electroid variety

A *circular planar electrical network* Γ is a finite graph embedded in a disk, with labeled boundary vertices and conductances $c_e > 0$.



Boundary voltages determine boundary currents:

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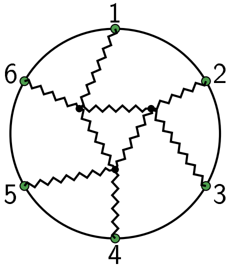

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Lam's embedding

- E_n denotes electrical networks modulo electrical equivalence.
- There is an embedding

$$\iota : E_n \hookrightarrow \mathrm{Gr}(n-1, 2n)_{\geq 0}.$$

- The *electroid variety* is the Zariski closure

$$\mathcal{X}_n = \overline{\iota(E_n)}^{\mathrm{Zar}}.$$

[Lam, 2018]

The grove algebra

- The *grove algebra* is the homogeneous coordinate ring

$$G_n := \mathbb{C}[\mathcal{X}_n] = \bigoplus_{m \geq 0} G_{m,n}.$$

- $\mathcal{X}_n \cong \mathrm{LG}(n-1, 2n-2) \Longrightarrow G_{m,n} \cong V_{m\omega_{n-1}}$ as an irreducible $\mathrm{Sp}(2n-2)$-module.

[Bychkov et al., 2023; Gao–Lam–Xu, 2025]

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Degree one: grove basis

$$\{L_\sigma : \sigma \in \mathrm{NC}(n)\} \text{ is a basis of } G_{1,n},$$

$$c \cdot L_\sigma = L_{\mathrm{Krew}(\sigma)},$$

equivalently, rotation of the associated noncrossing matching.

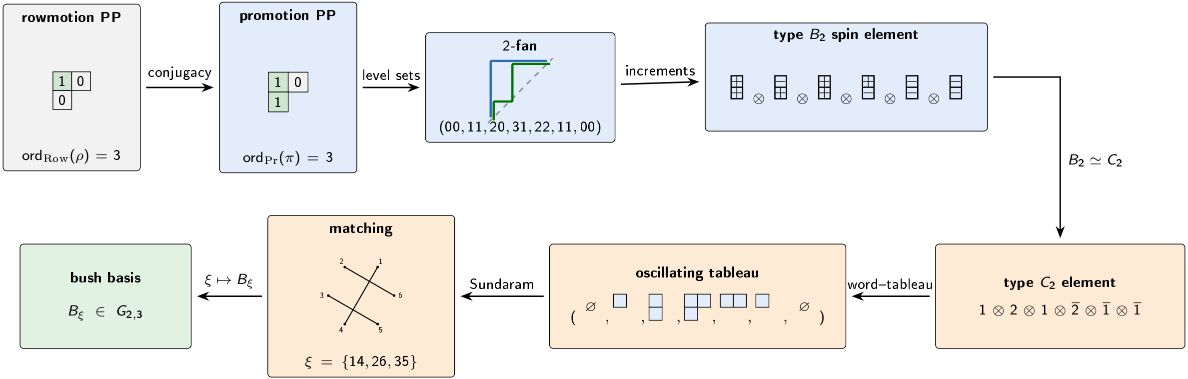
Degree two: bush basis

$$\{B_\xi : \xi \in \mathcal{TC}_3(2n)\} \text{ is a basis of } G_{2,n},$$

indexed by 3-noncrossing perfect matchings, and

$$c \cdot B_\xi = B_{\mathrm{rot}(\xi)}.$$

The running example: every map is equivariant



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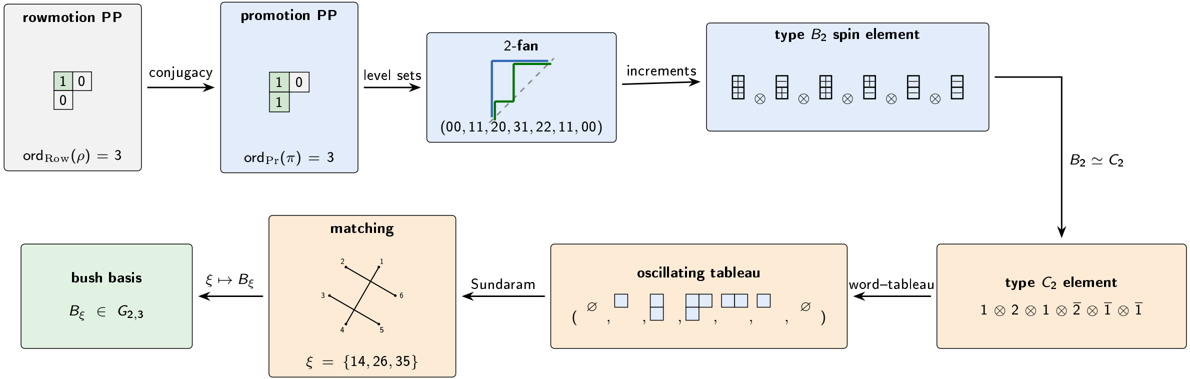

Consequently, in degree two, the promotion-equivariant bijection identifies staircase plane partitions with the bush indexing set. Therefore

$$\#\mathrm{Fix}(\mathrm{Pr}^k) = \#\mathrm{Fix}(\mathrm{rot}^k) = \mathrm{tr}(c^k \mid G_{2,n}),$$

and this trace is a symplectic-character value because $G_{2,n} \cong V_{2\omega_{n-1}}$.

Extracting cyclic sieving

The cyclic action rotates the bush basis, and the equivariant bijection identifies rotation with promotion. Thus, for $\zeta = e^{2\pi i/(2n)}$,

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Proctor's principal specialization [Proctor, 1990] gives

$$\mathrm{Sp}_{2n-2}(2\omega_{n-1}; q, q^2, \ldots, q^{n-1}) = q^{-2\binom{n}{2}} \prod_{1 \le i \le j \le n-1} \frac{1 - q^{i+j+4}}{1 - q^{i+j}}.$$

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Theorem (Hopkins–Kim–P, 26+)

For $n \ge 3$,

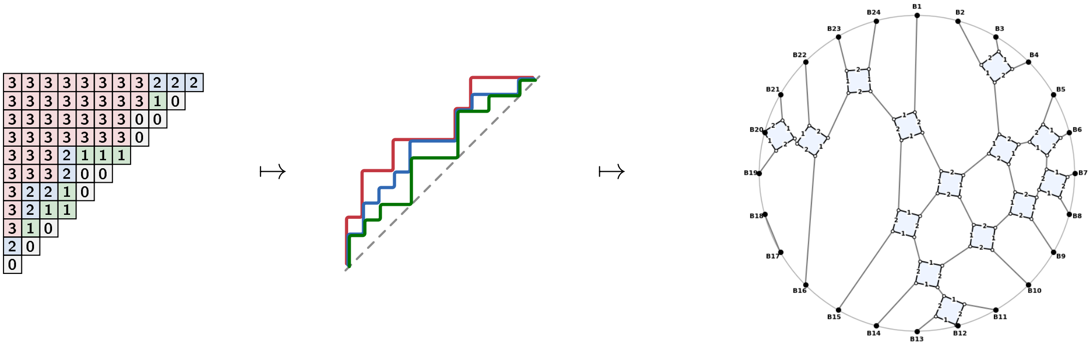
$$\left(\mathcal{PP}^2(\delta_{n-1}), \langle \mathrm{Pr} \rangle \simeq \mathbb{Z}/2n\mathbb{Z}, \Omega_{\delta_{n-1}}(2; q)\right)$$

exhibits CSP.

Where next?

Beyond height $m = 2$

- Construct a conjectural electrical canonical basis of $G_{m,n}$.
- Find a bijection between m-fans of Dyck paths and the basis elements that intertwines promotion and rotation.



Thank you!

AFTER EVERY GAME...
WE ROWMOTION!
ROWMOTION FOREVER!
ROW! ROW! ROW!
GO CATALAN NUMBERS!
Dyck path: N, E steps above y = x from (0,0) to (n,n)
UP-STEP #1
Catalan numbers
$C_n = \frac{1}{n+1}\binom{2n}{n}$
1, 1, 2, 5, 14, 42, ...
Rowmotion on J(P)
ρ : J(P) → J(P)
order ideals go brrr
DYCK FAN
TODAY'S THEOREM: EVERYTHING IS BEAUTIFUL?
COMBINATORICS IS LIFE
I LATTICES
SET BUILDER CLUB
POSETS
Promotion is cool... but have you tried ROWMOTION?
COMBINATORICS MAKES US MOVE!
NORTH, EAST, REPEAT!