

Tilepaint: visualizations and calculations in tilings, perfect matchings, and n -mer covers

Leigh Foster

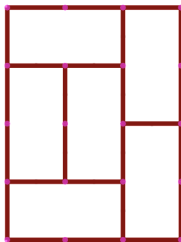
FPSAC 2026
University of Washington, Seattle

July 16, 2026

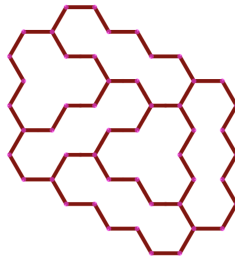
Tilings, perfect matchings, and n -mer covers



lozenge

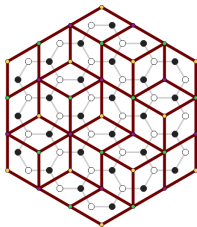


domino

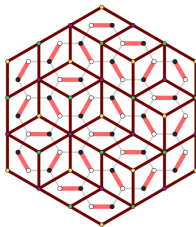
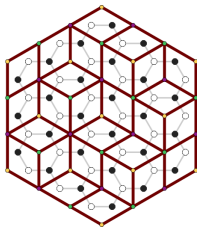


bones and stones

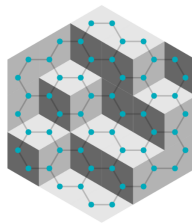
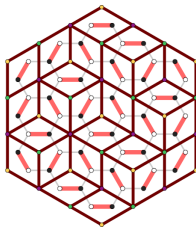
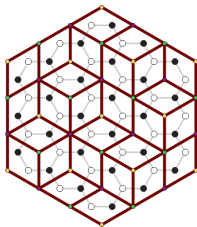
Tilings, **perfect matchings**, and n -mer covers



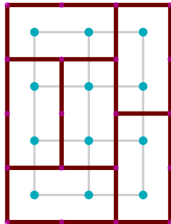
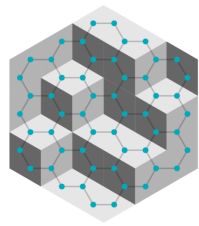
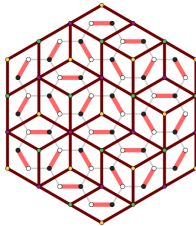
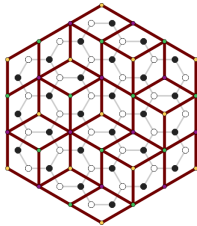
Tilings, **perfect matchings**, and n -mer covers



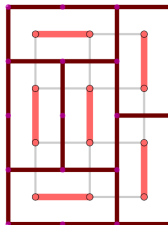
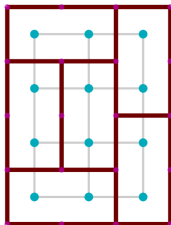
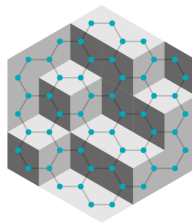
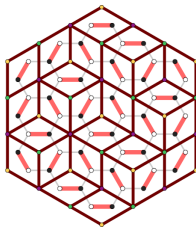
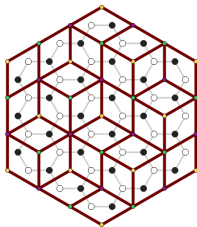
Tilings, **perfect matchings**, and n -mer covers



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Tilings, perfect matchings, and n -mer covers

Perfect matchings are also known as *dimer covers*.

Tilings, perfect matchings, and n -mer covers

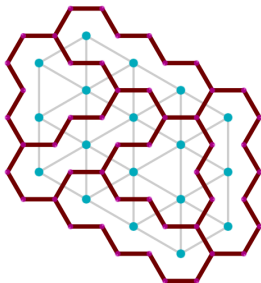
Perfect matchings are also known as *dimer covers*.

di- *two* -mer *parts*

Tilings, perfect matchings, and n -mer covers

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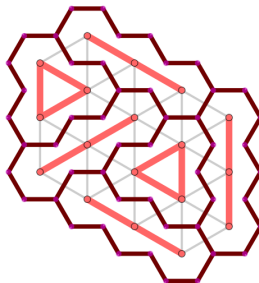
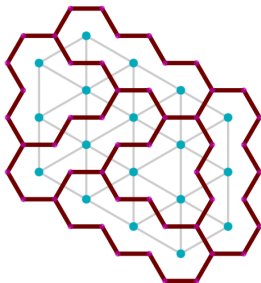
di- *two* -mer *parts*



Tilings, perfect matchings, and n -mer covers

Perfect matchings are also known as *dimer covers*.

di- *two* -mer *parts*



About Tilepaint

Tilepaint was created with the following goals in mind:

- accessibility
- no programming required
- integration with existing mathematical software
- immediate visualizations

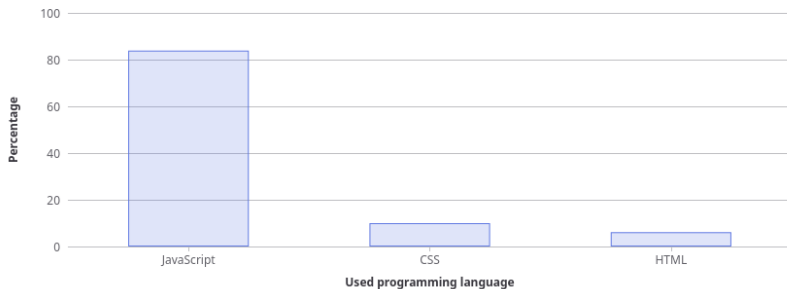
About Tilepaint

Leigh Foster / tilepaint / **Repository analytics**

Repository analytics

Programming languages used in this repository

Measured in bytes of code. Excludes generated and vendored code.



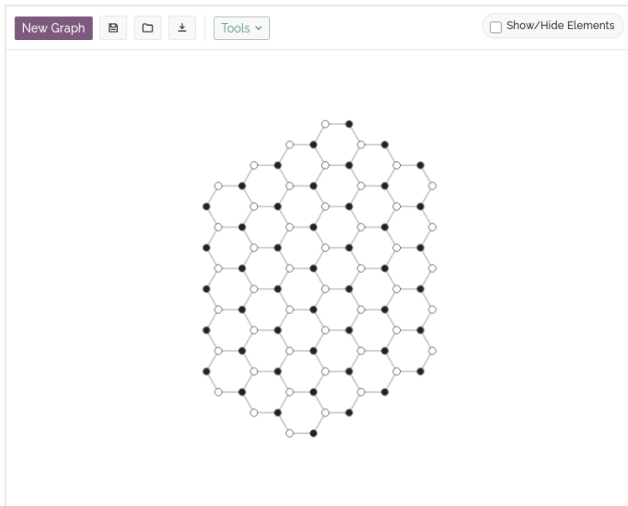
About Tilepaint

Open source, code available at
gitlab.com/mathematicleigh/tilepaint

Uses graph theory library [cytoscape.js](https://js.cytoscape.org) (js.cytoscape.org)

How do I use it?

Click an edge of the graph to make a tile. View options are found in "Show/Hide Elements", and various dimer/tiling tools are in the "Tools" menu. Have fun!



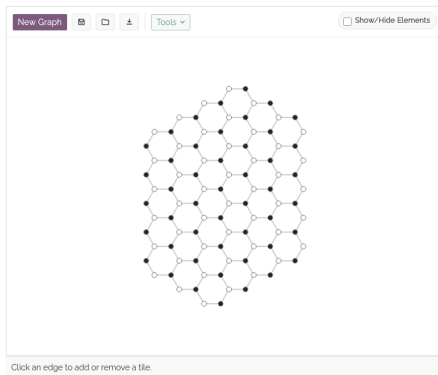
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mathematicleigh.com/tilepaint



Instructions

Click an edge of the graph to make a tile. View options are found in "Show/Hide Elements", and various dimer/tiling tools are in the "Tools" menu. Have fun!



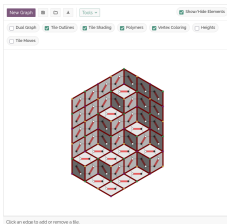
- Click on an edge of the graph!
- Try zooming in and out
- Click on more graph edges
- . . .

Instructions

Try changing the view: Click "Show/Hide Elements"

Instructions

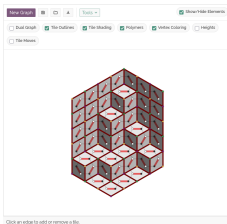
Try changing the view: Click "Show/Hide Elements"



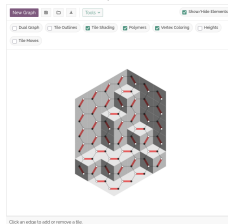
default view

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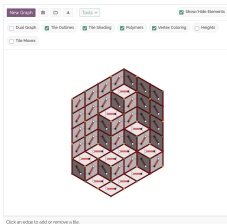
default view



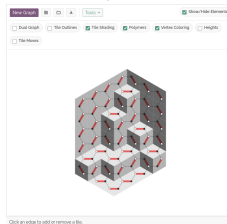
shading, polymers, vertex colors

Instructions

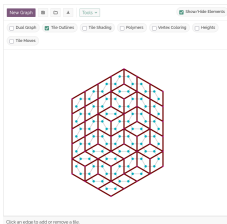
Try changing the view: Click "Show/Hide Elements"



default view



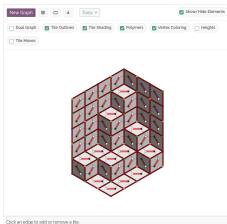
shading, polymers, vertex colors



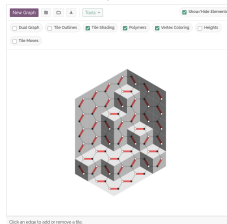
tile outlines only

Instructions

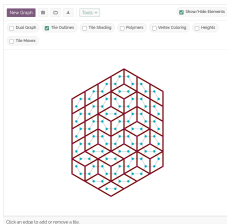
Try changing the view: Click "Show/Hide Elements"



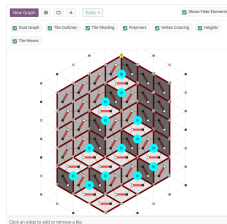
default view



shading, polymers, vertex colors



tile outlines only



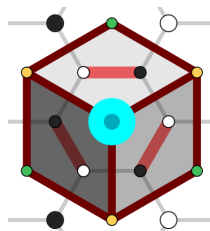
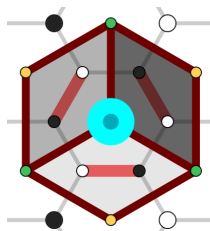
everything

Tile Moves (lozenges and dominos)

Each large cyan dot represents a possible tile move.

Tile Moves (lozenges and dominos)

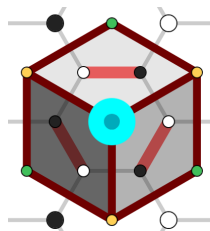
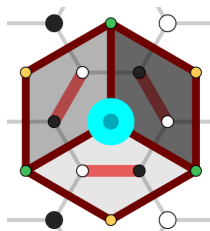
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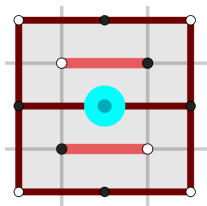
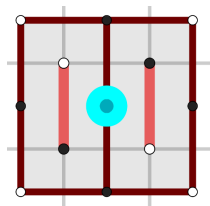
box (or benzene)
move

Tile Moves (lozenges and dominos)

Each large cyan dot represents a possible tile move.



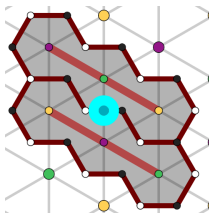
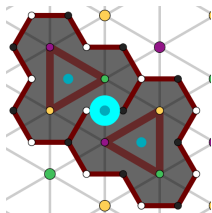
box (or benzene)
move



domino move

Tile Moves (stones and bones)

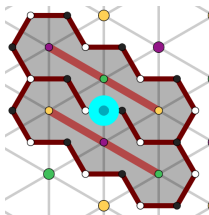
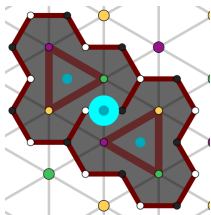
Each large cyan dot represents a possible *conjectural* tile move.
See Propp, twenty mostly open problems



← Swap two bones
for two stones

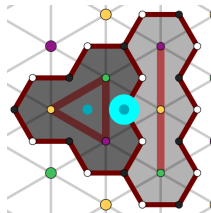
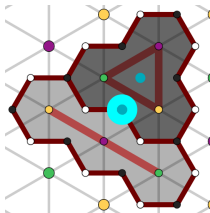
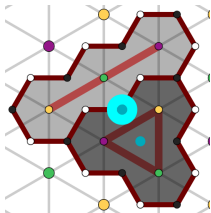
Tile Moves (stones and bones)

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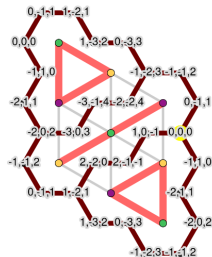
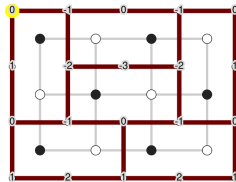
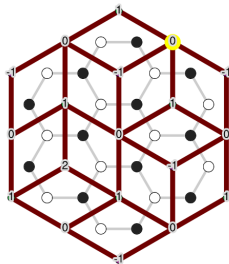
← Swap two bones
for two stones

↓ Rotate one bone
and one stone



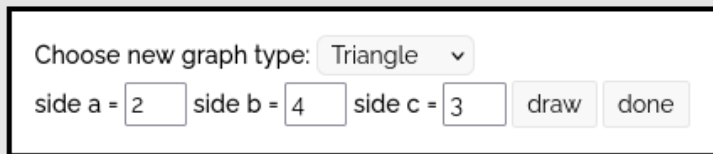
Height Function(s)

The “Heights” button shows, for lozenge and domino tilings, Thurston’s height function. For tilings of hexagons (triangular base grid), it shows a three-dimensional height function with coordinates (*orange*, *green*, *purple*).



New Graphs

Click the "New Graph" button in the top left corner to generate a new graph. You can choose between the three general grid types: triangular, hexagonal, and square grids. There is also a custom graph type available, discussed later.



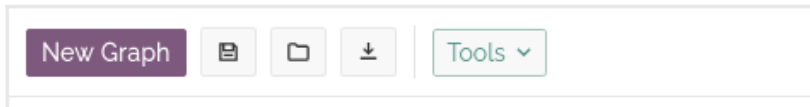
The image shows a dialog box titled "Choose new graph type:". It contains a dropdown menu currently set to "Triangle". Below the dropdown, there are three input fields for side lengths: "side a = 2", "side b = 4", and "side c = 3". To the right of these fields are two buttons labeled "draw" and "done".

Choose new graph type: Triangle ▾

side a = 2 side b = 4 side c = 3 draw done

Saving and Loading

The top bar shows three icons to the right of the “New Graph” button.



The save button saves the entire graph with all shown options. This exports a JSON file to save to your computer.



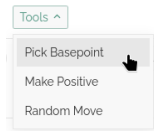
The load icon loads a JSON file previously exported from tilepaint.



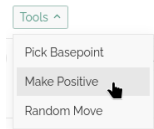
The last icon exports a JSON file with a list of all Conway-Lagarias words for the tiles in your tiling.

Tools

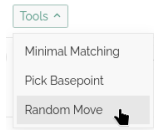
This menu has various options based on the shown graph type.



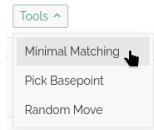
“Pick Basepoint” lets you click a vertex of a tile to set the basepoint (0-coordinate) for the height function.



“Make Positive” automatically shifts all shown height values to be non-negative.



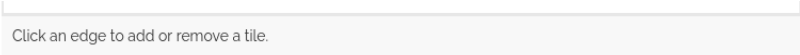
“Random Move” randomly performs a tile move, if available.



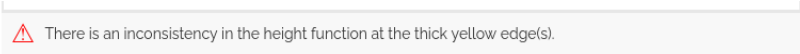
“Minimal Matching” draws the lozenge tiling corresponding to the empty room configuration (or plane partition will all entries 0).

Alert Bar

The bar at the bottom shows the text "Click an edge to add or remove a tile" when there are no errors in the graph.

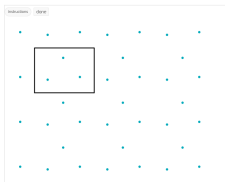
A light gray rectangular bar with a thin border. Inside, the text "Click an edge to add or remove a tile." is displayed in a dark gray font.

If an error exists (such as an inconsistent height function), the bar shows an alert with the relevant information.

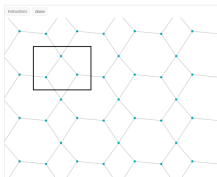
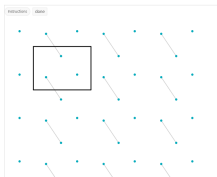
A light gray rectangular bar with a thin border. On the left, there is a red triangle icon with a white exclamation mark. To its right, the text "There is an inconsistency in the height function at the thick yellow edge(s)." is displayed in a dark gray font.

Custom Graphs

The black rectangle represents the fundamental domain of an infinite repeating graph.



Single click within the fundamental domain to add a vertex. Click twice on an existing vertex to remove it.



Single click one existing vertex then another (with at least one within the fundamental domain) to add an edge between them.

Custom Graphs



Note: your graph should have convex faces for tile drawing to work appropriately.

Click "done". Then, as with the other graph types, you can click an edge of the base graph to draw tiles and/or polymers.

