Séminaire Lotharingien de Combinatoire **XX** (2026)
Article #108, 6 pp.

Proceedings of the 38th Conference on Formal Power Series and Algebraic Combinatorics (Seattle)

Tilepaint: visualizations and calculations in tilings, perfect matchings, and n-mer covers

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Abstract. We introduce tilepaint, which is a web application written in JavaScript and using Cytoscape.js. It runs entirely in the user's browser, and works equally well on the computer or on mobile. It allows the user to explore perfect matchings, tilings, and n-mer (e.g. dimer) covers of a planar graph without any requisite knowledge of programming. It is also fully open-source under the AGPLv3 license, with source code available through gitlab.

1 Introduction

From tangrams to tessellations to brick pavers, we have many real-life examples of tilings of planar regions. The study of tilings and tilability has been of interest for hundreds of years and remains an active area of research. It is also an area of mathematics with easily described problems and intriguing visualizations, which allows us to reach a wider audience and engage students from a variety of backgrounds. We also have explicit relations between perfect matchings (or dimer covers) of the hexagon grid and lozenge tilings of the triangular lattice, and between perfect matchings on the square grid and domino tilings.

The program Tilepaint [6] allows the user to explore these concepts of tilings and n-mer covers on some preset graphs of a selectable size, and to create their own graph by drawing vertices and edges on a given fundamental domain. It can provide, with a single click, the word in the Conway tiling group [3] associated with each closed loop (or tile boundary), Thurston's height function [13], and the available set of tile moves (see [12] for examples with stones and bones, or [7] for the add/remove-a-box operation). It also allows the user to save and load graph states as JSON files, which are reasonably human-readable and can be imported into Python, SageMath, and other programming languages.

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1.1 Acknowledgements

Tilepaint is a rewrite and expansion of the software Dimerpaint [14], originally written in Python and Pygame by Ben Young, with later contributions by the author. The author would like to thank Hanna Mularczyk for helpful conversations, suggestions, and extensive bug testing.

2 The software

When the user first opens Tilepaint, they are greeted with the program's default $4 \times 3 \times 5$ hexagonal grid base graph. When an edge of the graph is clicked, the edge is highlighted and the corresponding tile is drawn (from the dual graph). This illustrates the bijection between dimers and lozenge tilings and shades the tiles to show how the dimer cover can be visualized as a stack of boxes. See Figure 3 for an example. The cyan dots in the center of each trio of tiles allow you to perform a 'box flip' – also called a benzene move in the language of SL_n webs and hourglass plabic graphs [8], or a tile move in the language of lozenge tilings. Tilepaint can automatically generate the minimal lozenge tiling in terms of Thurston's height function, and display these values at every vertex. This allows the user to observe how the height function changes dynamically as the tiling is modified.

2.1 Capabilities

The default hexagonal grid is only the first of the three built-in graph types, the other two being the triangular grid and the square grid. To create a new graph, the user clicks "New" and then can choose any of the three graph types, edit the size of the graph, and see an example of their choice before confirming. See Figure 1.

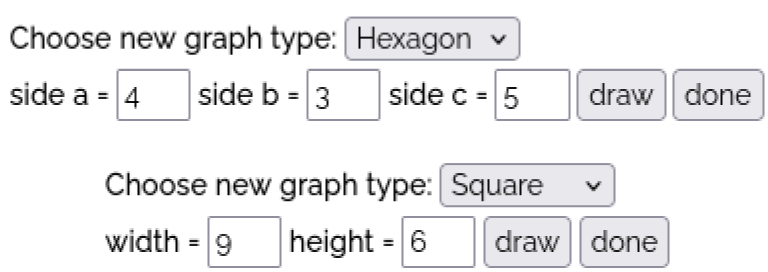

Figure 1: The new graph selection menu for the hexagonal grid (above) and square grid (below)

The second default graph type is the triangular grid. This allows the user to create trimer covers of the base grid, which includes stones and bones tilings as popularized by Conway and Lagarias [3]. These are an active area of research for the author and others. See [10, 1, 11] for some recent publications in the area, as well as [7, 4] for related work by the author. See Figure 2 for an example. Since the standard way of defining a height function fails for the triangular grid (and as such for stones and bones tilings), Tilepaint renders a three-dimensional height function at each vertex of the dual graph.

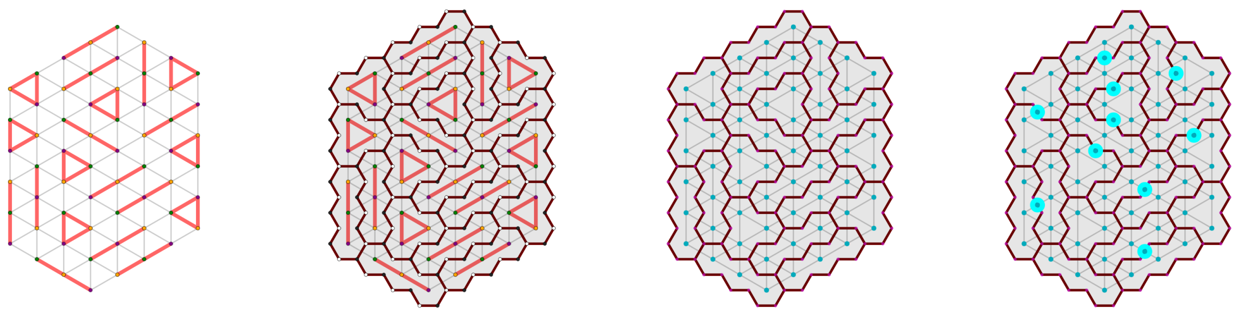

Figure 2: A tiling of the $4 \times 3 \times 5$ hexagonal region using stones and bones. From left to right: (1) the trimer cover of the triangular grid, (2) the trimer cover with its corresponding tiling, (3) the stones and bones tiling, and (4) the same tiling with conjectural tile moves shown [12]

Finally, we have similar examples on the square grid. A user can choose whether to generate their grid as a rectangle or as an Aztec diamond. In either case, each dimer cover of the graph is in bijection with a domino tiling on the dual graph. Such domino tilings have connections with statistical mechanics [2], alternating sign matrices [5], and limit shapes and the Arctic circle phenomenon [9].

2.2 Display options

Tilepaint also has many different view and drawing options. One particularly helpful set of configurations allows the user to see the connection between stacks of boxes and dimer covers, as mentioned above. See Figure 3. In general, the user can choose to display or hide the dual graph itself, the outline of each tile, the matched edges (or polymer configuration) on the base graph, the height function labeled on each vertex, and the set of available tile moves, when applicable. The vertices can also be displayed with the bi- or tripartite coloring, as appropriate.

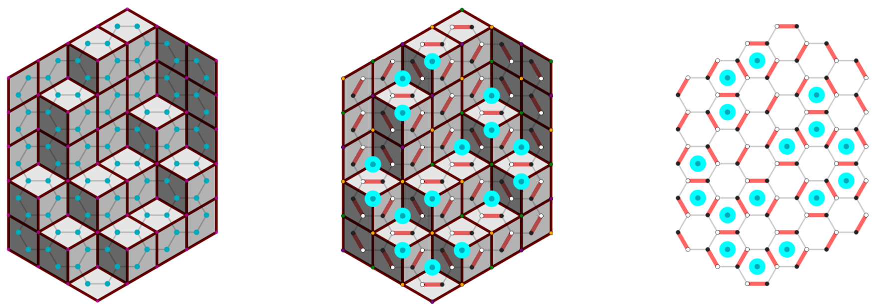

Figure 3: A visual representation of the bijection between box stacking and perfect matchings

2.3 Custom graph drawing

One other main feature of Tilepaint is custom graph drawing. The user is given a boxed fundamental domain, where they can click to add or remove vertices, and then choose two vertices to add edges. As soon as they click, each vertex or edge is repeated across the screen so that they can picture exactly what the final graph will be, as well as create edges that bridge across the fundamental domain with the rest of the graph. See Figure 4.

Once the user is satisfied, they click the “Done” button, at which point the dual graph is drawn (for a convex fundamental domain). At this point the graph is no longer editable, but the user can select edges to draw n-mer covers or tilings, as in the case of the other graph types.

3 Accessibility

A central goal of Tilepaint is to make serious mathematical software accessible to users with little to no programming experience. The study of tilings and tilability has many easily described problems, as well as engaging pictures and figures, allowing us to reach a wider audience and provide an avenue for students from a variety of backgrounds to engage with meaningful mathematical research. Not only does using JavaScript allow for a clean, easy-to-access user interface, it also has the advantage of widely available documentation and tutorials for new programmers, providing students interested in contributing an opportunity to do so without having to learn a new programming language.

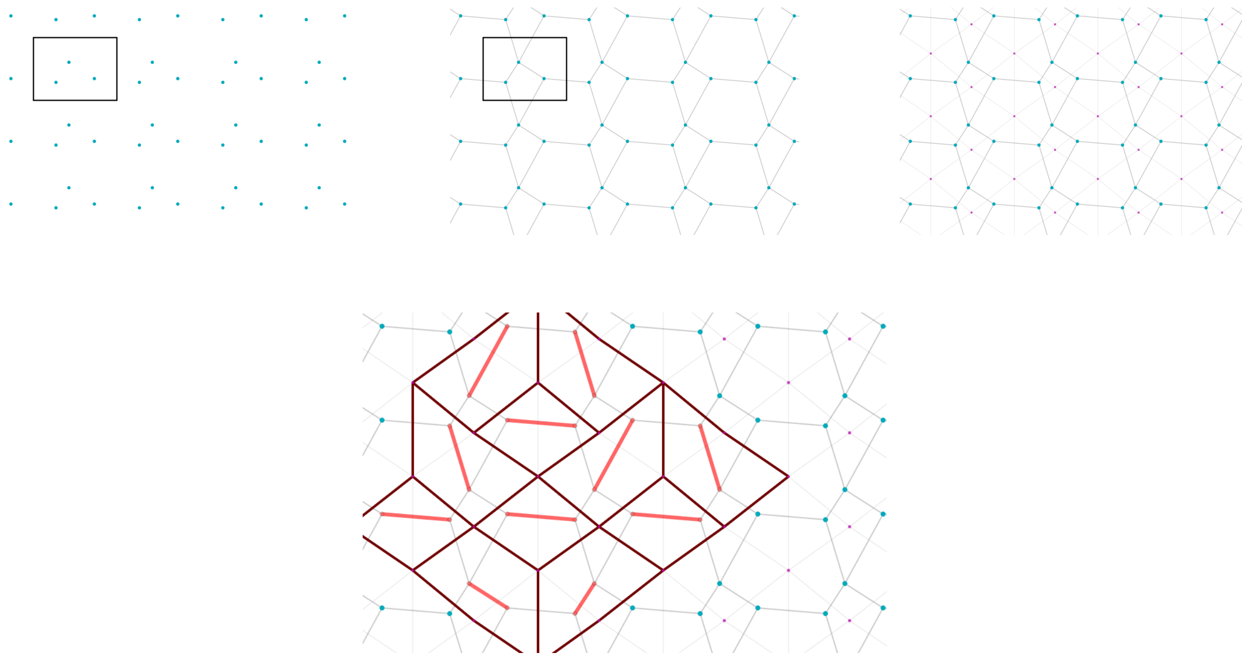

Figure 4: From left to right: vertices added to the fundamental domain, then edges, then the resulting graph after clicking "Done". Below: a sample matching/tiling of a region of the created graph

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