DIFFERENTIAL GEOMETRY/PDE SEMINAR

WEDNESDAY, FEBRUARY 17
COMMUNICATIONS 326
3:45 PM

Spike-layer solutions of some singular perturbation elliptic problems

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We consider a class of nonlinear elliptic problems of the form

$$\varepsilon^2 \Delta u - u + f(u) = 0 \quad \text{in } \Omega,$$

where the simplest model for the nonlinearity f is given by $f(t) = t^p$ with $1 < p < \frac{N+2}{N-2}$ if $N \geq 3$, and $1 < p < +\infty$ otherwise. Here Ω is a bounded domain with smooth boundary, and $\varepsilon > 0$ is a small parameter. We are interested in positive solutions to this equation satisfying zero Dirichlet or Neumann boundary conditions on $\partial\Omega$. We study the behavior of *least energy solutions* as ε goes to zero. These are known to develop a sharp spike shape located respectively around a *most centered* point of the domain or a *most curved* point of its boundary. Elementary proofs of these facts are found, which additionally lift certain strong nondegeneracy assumption commonly required in the literature. The approach is flexible enough to apply to related problems where such assumptions are virtually uncheckable.