DIFFERENTIAL GEOMETRY/PDE SEMINAR

Wednesday, January 13
Communications 326
3:45 pm

Propagation of singularities and the structure of the scattering matrices in many-body scattering

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In this talk I will discuss the propagation of singularities of generalized eigenfunctions of a many-body Hamiltonian $H = \Delta + V$, $V = \sum_a V_a$, under the assumption that the two-body interactions V_a are real-valued polyhomogeneous symbols of order -1 (e.g. Coulomb-type with the singularity at the origin removed). In particular, I will explain the propagation results in three-body scattering, and also in many-body scattering under the additional assumption that no subsystem has a bound state. These results provide a connection between quantum mechanics and classical mechanics which is very similar to the one between wave propagation and geometric optics that appears, for example, in the works of Hörmander, Melrose, Sjöstrand and Lebeau. They also prove that the wave front relation of the free-to-free S-matrix (which is all of the S-matrix if no subsystem has a bound state) is given by the broken geodesic flow, broken at the 'singular directions', on $\mathbb{S}^{n-1}$ at time π. I also extend these results to the case when X is a manifold with boundary equipped with a scattering metric g, a cleanly intersecting family $\mathcal{C}$ of closed embedded submanifolds of ∂X which are locally linearizable, Δ the Laplacian of g and $V \in \mathcal{C}^\infty([X;\mathcal{C}])$ vanishes in the nonsingular directions, i.e. at $\partial X \setminus \bigcup\{C \in \mathcal{C} :\ C \neq \partial X\}$.