# DIFFERENTIAL GEOMETRY/PDE SEMINAR

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Communications 326
3:45 pm

## The Hamilton-Jacobi equation and variational analysis

Terry Rockafellar
(University of Washington)

The Hamilton-Jacobi equation is a first-order nonlinear PDE which goes back to the classical calculus of variations, where it arose in characterizing "value functions" as solutions to certain Cauchy problems. Value functions describe a kind of propagation and they are nowadays closely connected with feedback rules in optimal control, but for inescapable reasons they often lack differentiability. The PDE must therefore be posed in a generalized sense, but it has not been easy to find the right way to do this.

A natural approach might be to focus on an almost everywhere version of the PDE in terms of Lipschitz continuous functions as potential solutions, but uniqueness of solutions then fails. This difficulty has lead to the development of stricter concepts like that of "viscosity" solutions. In such terms, existence and uniqueness theorems have indeed been proved, but they still fall short because the assumptions they rely on exclude many of the value functions one is seeking to characterize.

In fact there is a strong incentive for treating, as unique solutions to a generalized Hamilton-Jacobi equation, value functions that are merely lower semicontinuous and can take on infinite values. That is now possible through notions of variational analysis which will be explained in this talk.