DIFFERENTIAL GEOMETRY/PDE SEMINAR

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Factorization of second order elliptic operators, complete systems of exact solutions and other applications

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It is a well known fact that given a nonvanishing particular solution for a one dimensional stationary Schrödinger equation, the Schrödinger operator can be factorized into a pair of linear first order differential operators and consequently solved. We show an analogous result in n dimensions. In the talk we give a detailed analysis of the two-dimensional situation in which the factorizing first order operators represent Vekua operators (of a special form) whose null solutions are known as generalized analytic or pseudoanalytic functions. As one of the corollaries we construct a Vekua equation possessing the following special property. The real parts of its solutions are solutions of the original bidimensional stationary Schrödinger equation

$$(\operatorname{div} p \operatorname{grad} + q)u = 0$$

and the imaginary parts are solutions of an associated Schrödinger equation with a potential having the form of a potential obtained after the Darboux transformation. We give explicit formulas for reconstructing imaginary parts by their real counterparts and vice versa generalizing in this way the well known in complex analysis procedure for constructing conjugate harmonic functions. Moreover, using L. Bers theory of Taylor series in formal powers for pseudoanalytic functions we obtain a complete (in C-norm) system of solutions of the original Schrödinger equation which we construct explicitly for an ample class of Schrödinger equations when a so called condition S introduced in [1,2] is fulfilled. For example it is possible, when the coefficients are functions of one Cartesian, spherical, parabolic or elliptic variable

and in many other cases. We give examples of application of the proposed procedure for obtaining a complete system of solutions of the Schrödinger equation. The procedure is algorithmically simple and can be implemented with the aid of a computer system of symbolic or numerical calculation. Our results are applicable to other equations of mathematical physics as, e.g., the conductivity equation and the Dirac equation in a two-dimensional situation. In the case of the conductivity equation we construct in explicit form a complete system of solutions when the conductivity of a medium fulfills the condition S mentioned above.

References:

1. V. V. Kravchenko On the reduction of the multidimensional stationary Schrödinger equation to a first order equation and its relation to the pseudoanalytic function theory. Journal of Physics A: Mathematical and General, 2005, v. 38, No. 4, 851-868.

2. V. V. Kravchenko On a relation of pseudoanalytic function theory to the two-dimensional stationary Schrödinger equation and Taylor series in formal powers for its solutions. Journal of Physics A: Mathematical and General, 2005, v. 38, No. 18, 3947-3964.