Factorization of second order elliptic operators, complete
systems of exact solutions and other applications

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It is a well known fact that given a nonvanishing particular solution for a
one dimensional stationary Schrödinger equation, the Schrödinger operator
can be factorized into a pair of linear first order differential operators and
consequently solved. We show an analogous result in n dimensions. In the
talk we give a detailed analysis of the two-dimensional situation in which the
factorizing first order operators represent Vekua operators (of a special form)
whose null solutions are known as generalized analytic or pseudoanalytic
functions. As one of the corollaries we construct a Vekua equation possessing
the following special property. The real parts of its solutions are solutions of
the original bidimensional stationary Schrödinger equation

$$(\text{div } p \text{ grad } +g)u = 0$$

and the imaginary parts are solutions of an associated Schrödinger equation
with a potential having the form of a potential obtained after the Darboux
transformation. We give explicit formulas for reconstructing imaginary parts
by their real counterparts and vice versa generalizing in this way the well
known in complex analysis procedure for constructing conjugate harmonic
functions. Moreover, using L. Bers theory of Taylor series in formal powers
for pseudoanalytic functions we obtain a complete (in C-norm) system of
solutions of the original Schrödinger equation which we construct explicitly
for an ample class of Schrödinger equations when a so called condition S
introduced in [1,2] is fulfilled. For example it is possible, when the coeffi-
cients are functions of one Cartesian, spherical, parabolic or elliptic variable
and in many other cases. We give examples of application of the proposed procedure for obtaining a complete system of solutions of the Schrödinger equation. The procedure is algorithmically simple and can be implemented with the aid of a computer system of symbolic or numerical calculation. Our results are applicable to other equations of mathematical physics as, e.g., the conductivity equation and the Dirac equation in a two-dimensional situation. In the case of the conductivity equation we construct in explicit form a complete system of solutions when the conductivity of a medium fulfills the condition S mentioned above.

References:

1. V. V. Kravchenko On the reduction of the multidimensional stationary Schrödinger equation to a first order equation and its relation to the pseudoanalytic function theory. Journal of Physics A: Mathematical and General, 2005, v. 38, No. 4, 851-868.


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