DIFFERENTIAL GEOMETRY/PDE/PROBABILITY SEMINAR

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Volume Growth, Brownian motion, and Stochastic Completeness of a Complete Riemannian manifold

Elton Hsu
(Northwestern University)

A geodesically complete Riemannian manifold is called stochastically complete if its heat kernel (the minimal fundamental solution of the parabolic Laplace-Beltrami operator) is integrated to one. Since the heat kernel is the transition density function of Riemannian Brownian motion, a manifold is stochastically complete if and only if Brownian motion does not explode. To find a proper geometric condition for stochastic completeness is an old geometric problem. The first result in this direction was due to S. T. Yau, who proved that a Riemannian manifold is stochastically complete if its Ricci curvature is bounded from below by a constant. It has been know for quite some time that the property of stochastic completeness is intimately related to the volume growth of a Riemannian manifold. We study stochastic completeness by looking at the more refined question of upper escaping rates of Riemannian Brownian motion. We show how the Neumann heat kernel, time reversal of reflecting Brownian motion, and volumes of geodesic balls come together and give an elegant and often sharp upper bound of the escaping rate solely in terms of the volume growth function without any extra geometric restriction. This is a joint work with Guang Nan Qin of Institute of Applied Mathematics of the Chinese Academy of Sciences.

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