DIFFERENTIAL GEOMETRY SEMINAR

Monday, May 8
Communications 226
3:45 p.m.

Fredholm Operators On Asymptotically Hyperbolic Manifolds

Jack Lee
(University of Washington)

The most important applications of partial differential equations to differential geometry involve elliptic operators acting on sections of vector bundles. The applications usually require showing that an operator P is Fredholm: its kernel is finite-dimensional, and its image is a finite-codimensional closed subspace of the range (usually in some Sobolev or Hölder norm). From this it follows by standard functional-analytic techniques that the equation $Pu = f$ can be solved if and only if f is orthogonal to $\operatorname{Ker} P^*$. Although every elliptic operator on a compact manifold is Fredholm, in the noncompact case the question is much more subtle.

In this talk I will discuss some recent results concerning Fredholm properties of Laplace-type operators on *asymptotically hyperbolic* manifolds: these are interiors of compact manifolds-with-boundary, endowed with complete metrics whose sectional curvatures approach -1 at a specified rate near the boundary. The natural settings for studying such operators are Sobolev or Hölder spaces weighted by a power of a smooth positive defining function for the boundary. By examining the asymptotic behavior near infinity, one can see easily that for each family of Sobolev or Hölder spaces, there is only a certain bounded range of weights for which such an operator can possibly be Fredholm.

I will discuss the ideas involved in the proof of the following theorem: *Any Laplace-type operator acting on symmetric or antisymmetric tensors is Fredholm on weighted L^2 Sobolev spaces and weighted Hölder spaces for exactly those weights allowed by the asymptotic analysis.* If I can finish working out the details this week, I hope also to be able to announce a proof of the analogous result for L^p Sobolev spaces. Applications include an elementary proof of Mazzeo's Hodge theorem for asymptotically hyperbolic manifolds; the existence of Einstein metrics with prescribed conformal infinity; and new existence and regularity results for the constraint equation for initial data in Einstein's gravitation equation.

For more information about this seminar, view the Differential Geometry Seminar World-Wide Web page at

`http://www.math.washington.edu/~lee/Seminars/dg.html`