DIFFERENTIAL GEOMETRY SEMINAR

MONDAY, MAY 15
COMMUNICATIONS 226
3:45 P.M.

Pseudo-Einstein Structures On CR Manifolds

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A *CR manifold* M is an abstract model of a real hypersurface in $\mathbf{C}^n$. We would like to study curvature and other structures on M, as in the Riemannian case. But rather than impose a Riemannian metric on M, we wish to somehow define curvature that relates to the intrinsic structure of M. (In particular, we can base this curvature on the *Levi form*, a Hermitian bilinear form that will take the place of a Riemannian metric in much of what we do. A choice of Levi form defines a *pseudohermitian structure* on M; since the CR structure only determines the Levi form up to a conformal multiple, we may consider different pseudohermitian structures on a single CR manifold.) Following Webster and Tanaka, we define the *pseudohermitian curvature tensor* on M, with corresponding Ricci tensor and scalar curvature. We may now ask questions similar to those for a Riemannian manifold. For instance, can we find a choice of pseudohermitian structure that results in constant pseudohermitian scalar curvature? (This is the Yamabe problem for CR manifolds.) The question we will attempt to deal with is: what is the analogue of an Einstein metric on CR manifolds, and when does such an analogue exist?

We say M is *pseudo-Einstein* if the pseudohermitian Ricci tensor is a scalar multiple of the Levi form. (Here the source of the terminology is clear: a Riemannian metric is Einstein if the induced Ricci tensor is a scalar multiple of the metric. Note that the Levi form plays the role of the metric.) Following work of J. Lee, we will show that such a manifold satisfies a topological condition, and we will discuss the conjecture that the existence of a pseudo-Einstein structure is, in fact, equivalent to this topological condition. If time permits, we will discuss a class of examples (due to S. Dragomir) that satisfy this conjecture but do not follow from known cases of the conjecture.

For more information about this seminar, view the Differential Geometry Seminar World-Wide Web page at

`http://www.math.washington.edu/~lee/Seminars/dg.html`