Seattle Noncommutative Algebra Day

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ABSTRACT

Algebraic structures in group-theoretical fusion categories

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Introduced by Pavel Etingof, Dmitri Nikshych, and Victor Ostrik (2005), "group-theoretical fusion categories" (GTFCs) are a certain kind of monoidal category whose construction depends on group-theoretic data. They are a vital part of the classification program of general fusion categories, and due to their explicit construction, they also serve as a go-to testing ground for results about fusion categories. The goal of this talk is to present results on the representation theory of GTFCs, which by work of Ostrik (2003), boils down to understanding algebraic structures in GTFCs. The main result is a construction of explicit Morita equivalence class representatives of indecomposable, separable algebras in GTFCs. Background on monoidal categories will be provided, along with interesting auxiliary results appearing in our work. This is joint work with Yiby Morales, Monique Müller, Julia Plavnik, Ana Ros Camacho, and Angela Tabiri.

On the Noether bound for noncommutative rings

Ellen Kirkman

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Let $\mathbb{k}$ be a field of characteristic zero. In 1915 E. Noether proved that if G is a finite group acting naturally on a polynomial ring $\mathbb{k}[x_1, \ldots, x_n]$, then the degrees of a minimal set of generators of the subring of invariants are bounded above by the order of the group, and in 2011 P. Symonds proved that for general $\mathbb{k}$, an upper bound is $n(|G|-1)$ when $n \geq 2$ and $|G| > 1$. Replacing $\mathbb{k}[x_1, \ldots, x_n]$ by an Artin-Schelter regular algebra A, we obtain examples (in characteristic zero) where the Noether bound on the degrees of a minimal set of generators of A^G does not hold, and discuss the problem of finding bounds on the degrees of minimal generators of A^G.

Realization of PBW-deformations of type A_n quantum groups via multiple Ore extensions

Dingguo Wang

Qufu Normal University

The notion of multiple Ore extension is introduced as a natural generalization of Ore extensions and double Ore extensions. For a PBW-deformation $\mathfrak{B}_q(\mathfrak{sl}(n+1,\mathbb{C}))$ of type A_n quantum group, we explicitly obtain the commutation relations of its root vectors, then show that it can be realized via a series of multiple Ore extensions, which we call a ladder Ore extension of type $(1, 2, \cdots, n)$. Moreover, we analyze the quantum algebras $\mathfrak{B}_q(\mathfrak{g})$ with $\mathfrak{g}$ of type D_4, B_2 and G_2 and give some examples and counterexamples that can be realized by a ladder Ore extension. This is joint work with Yongjun Xu and Hualin Huang.

Admissible D-modules over symmetric spaces and cyclic quivers

Toby Stafford

University of Manchester

Let G be a complex reductive Lie group with Lie algebra $\mathfrak{g} = Lie(G)$. The famous regularity theorem of Harish-Chandra determines the structure of G-equivariant eigendistributions on a real form $\mathfrak{g}_0$ of $\mathfrak{g}$. Algebraically this boils down to understanding a particular module, called the Harish-Chandra $\mathcal{D}$-module N, over the ring of differential operators $\mathcal{D}(V)$ on $V = \mathfrak{g}$. Harish-Chandra's theorem simply states that

N has no factor that is d-torsion, where d denotes the discriminant. Hotta and Kashiwara further showed that N is even semi-simple and this in turns has significant applications to the geometric theory of $\mathfrak{g}$-representations.

Various people have looked at generalisations of these theorems to more general representations of G, in particular to symmetric spaces V, with both positive and negative results. In this talk we help to explain this. First, in work with Levasseur we have shown that there is a natural map from the invariant ring $(\mathcal{D}(V))^G$ onto the (spherical) Cherednik algebra H. (This is a deformation of $(\mathcal{D}(h))^W$ for the appropriate representation h of a complex reflection group W.) An analogue of the Harish-Chandra module N and the discriminant d is defined here and we show that: *If H is a simple algebra then N has no factor, or submodule, that is d-torsion.*

Secondly we show N *is a semi-simple* $\mathcal{D}(V)$*-module if and only if the Hecke algebra associated to* H *is a semi-simple algebra.*

The module N is an example of an admissible $\mathcal{D}(V)$-module and these results have applications to other admissible $\mathcal{D}(V)$-modules. (joint with Bellamy and Nevins)

Relative homological theory and Hopf algebra actions

Xiuli Chen

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Firstly, let H be a finite dimensional Hopf algebra and A be an algebra over a base field k. We discuss the cotorsion dimension of the smash product $A\#H$. We prove that $l.cot.D(A\#H) \leq l.cot.D(A) + rD(H)$ which generalizes a result in group ring. Moreover, we give some sufficient conditions such that $l.cot.D(A\#H) = l.cot.D(A)$. As applications, we study the invariants of IF properties and Gorenstein global dimensions. Secondly, it is proved that the left FP-projective dimension is invariant under cleft extensions when H is semisimple and A is left coherent. Finally, we discuss cotorsion dimension relative to semidualizing modules. Let C be a semidualizing R-module, where R is a commutative ring. We introduce the definition of C-cotorsion modules, and obtain the properties of C-cotorsion modules. As applications, we give some new characterizations for perfect rings. We also study the Foxby equivalences between the subclasses of the Auslander class and that of the Bass class with respect to C.

On the Hochschild cohomology ring of some twisted tensor products of algebras

Pablo Ocal

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The (ring structure of the) Hochschild cohomology of the tensor product of two algebras was understood better thanks to Le and Zhou, who were able to express it in terms of the two algebras. Using work by Grimley, Nguyen, and Witherspoon, as well as homotopy lifting techniques for Gerstenhaber brackets introduced by Volkov, we generalize Le and Zhou's result to some twisted tensor products. These have important applications in some quantum complete intersections also studied by Lopes and Solotar.