# SNAD IV

**October 30, 2016**

# ABSTRACT

## Kodaira dimension of Orders

**Colin Ingalls**

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Joint work with Nathan Grieve. The Kodaira dimension is the coarsest birational invariant. We present several approaches to the Kodaira dimension of an order and discuss to what extent these are equivalent and invariant under the choice of order. We also show that if we have two division algebra D1 inside D2 with the centres satisfying Z1 inside Z2, both finite over their centres, then the Kodaira dimension can only increase.

## On the category of coherent sheaves over weighted projective lines

**Jianmin Chen**

Xiamen University, China

In this talk, I will report some of my recent joint work on the category of coherent sheaves over weighted projective lines. We prove that for each tilting bundle T (with endomorphism algebra A) over the weighted projective line of type (2,2,n), the missing part, from the category of coherent sheaves to the category of finitely generated right A-modules, carries the structure of an abelian category. We investigate tilting objects in the stable category of vector bundles over a weighted projective line of type (2,2,2,2), and then classify all the endomorphism algebras of tilting objects in the category of coherent sheaves. Moreover, we establish a closed link between the category of coherent sheaves over a weighted projective line of tubular type and that on an elliptic curve.

# Deformations of $W_{A,D,E}$ Superconformal Field Theories


## Emily Nardoni

University of California, San Diego, USA


The simply-laced Lie groups $A_k = SU(n+1), D_k = SO(2k), E_{6,7,8}$ classify far-flung things physical mathematics. In this talk, I'll focus on one particular realization of this $ADE$ classification in physics: superconformal quantum field theories with superpotentials given by Arnold's A,D,E simple surface singularities ("$W_{A,D,E}$"). I'll begin by reviewing the well-studied 2d Landau-Ginzburg theories with $N = 2$ supersymmetry and $W_{A,D,E}$ superpotentials. Our main focus, however, will be on 4d $N = 1$ variants of supersymmetric Quantum Chromodynamics (SQCD), where matter fields that transform in the adjoint representation of the $SU(N)$ or $U(N)$ gauge group appear in $W_{A,D,E}$ superpotentials. In this context, novelties arise due to the matrix-variable adjoint fields, especially with regards to the noncommuting structure of the chiral ring. I'll describe recent attempts to explore these issues by considering various deformations of these 4d $W_{A,D,E}$ theories, and the resulting renormalization group flows.

# Noncommutative minimal surfaces


## Sue Sierra

University of Edinburgh, UK


In the classification of (commutative) projective surfaces, one first classifies minimal models for a given birational class, and then shows that any surface can be blown down at a finite number of curves to obtain a minimal model. Artin has proposed a similar programme for noncommutative surfaces (that is, domains of GK-dimension 3). In the generic "rational" case of rings birational to a Sklyanin algebra, the likely candidates for minimal models are the Sklyanin algebra itself and Van den Bergh's quadric surfaces. We show, using our previously developed noncommutative version of blowing down, that these algebras are minimal in a very strong sense: given a Sklyanin algebra or quadric $R$, if $S$ is a connected graded, noetherian overring of $R$ with the same graded ring of fractions, then $S = R$. This is joint work with Rogalski and Stafford.

# An abstract characterization of noncommutative $\mathbb{P}^1$-bundles

**Adam Nyman**

Western Washington University, USA

The notion of noncommutative $\mathbb{P}^1$-bundle was discovered and studied by M. Van den Bergh. Examples include noncommutative ruled surfaces, noncommutative Del Pezzo surfaces, and noncommutative curves of genus zero. In this talk we describe necessary and sufficient conditions for a $k$-linear abelian category to be equivalent to a noncommutative $\mathbb{P}^1$-bundle over a pair of division rings. We then present two consequences: Piontkovski's noncommutative projective lines are noncommutative $\mathbb{P}^1$-bundles, and noncommutative symmetric algebras are graded coherent.

# On the cohomology of Doi-Hopf modules

**Hong Zhu**

Changzhou University, China

Let $H$ be a Hopf algebra over a field $k$, and $(H, A, C)$ be a Doi-Hopf datum. The category of comodules over $\mathcal{C}$ and Doi-Hopf modules are then Grothendieck categories with enough injectives. We study the derived functors of the associated $Hom$ functors, and of the coinvariant functor, and discuss spectral sequences that connect them. We also discuss when the coinvariants functor preserves injectives.

# Singularity categories of deformations of Kleinian singularities

**Simon Crawford**

University of Edinburgh, UK

The Kleinian singularities make up a family of well-understood (commutative) surface singularities. In 1998, Crawley-Boevey and Holland introduced a family of algebras which may be viewed as noncommutative deformations of Kleinian singularities. Using singularity categories, I will make comparisons between the types of singularity arising in the commutative and noncommutative settings. I will also show that the "most singular" of these noncommutative deformations has a noncommutative resolution for which an analogue of the geometric McKay correspondence holds.