Twists of graded Poisson algebras and related properties

Xingting Wang

Howard University

Joint with Xin Tang and James J. Zhang

A Conference in Honor of S. Paul Smith on the occasion of his 65th Birthday

June. 24, 2020

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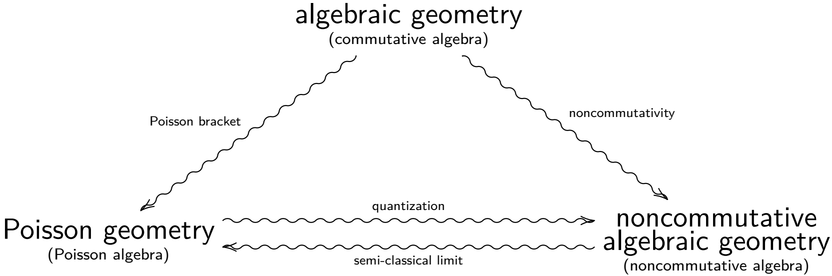

Setup

- $\mathbb{k}$: base field

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- Poisson algebra A: commutative $\mathbb{k}$-algebra with Poisson bracket

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that is (1) Lie bracket and (2) biderivation.

- graded Poisson algebra $A = \Bbbk[x_1, \ldots, x_n]$: multiplication and bracket both graded.

Sierra's Poisson algebra $A(n, a)$

Definition (Lecoutre-Sierra, 19)

Set $n \geq 1$ and $a \in \mathbb{k}$. Set $A(n, a) := \mathbb{k}[x_0, \dots, x_n]$ with

$$\boxed{\{x_i, x_j\} := (a + j)x_{i-1}x_j - (a + i)x_{j-1}x_i.}$$

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- $A(1, a) = \Bbbk[x_0, x_1]$ with $\{x_0, x_1\} = -ax_0^2$

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$$\begin{aligned}
\{x_0, x_1\} &= -ax_0^2, \\
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- $A(3, -\frac{5}{4})$ is Pym's exceptional Poisson algebra $E(3)$

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- Poisson derivation $\Rightarrow$ semi-Poisson derivation $\Rightarrow$ derivation

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The twist of (A, π) is (A^δ, π_{new}) such that

- $A = A^\delta$ as commutative algebras
- $\pi_{new} = \pi + E \wedge \delta$ or

$$\boxed{\{a, b\}_{new} = \{a, b\} + E(a)\delta(b) - \delta(a)E(b)}$$

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- $\Delta := x_0 \frac{\partial}{\partial x_1} + \cdots + x_{n-1} \frac{\partial}{\partial x_n}$: downward Poisson derivation
- $A(n, a)^{b\Delta} \cong A(n, a - b)$

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Definition (Tang-Zhang-W. 22)

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- A is rigid if $rgt(A) = 0$ and -1 rigid if $rgt(A) = -1$

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Definition

The modular derivation $\mathfrak{m}$ of A is

$$\boxed{\mathfrak{m}(a) := \mathrm{Div}(H_a).}$$

A is called unimodular if $\mathfrak{m} = 0$.

Remark

$\mathfrak{m} \in Pder(A)$ *and* $\mathrm{Div}(\mathfrak{m}) = 0$

Modular derivation

Lemma (Tang-Zhang-W. 22)

- $A = \Bbbk[x_1, \cdots, x_n]$: *$\mathbb{Z}$-graded Poisson polynomial algebra*

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- $\delta \in Pder(A)$
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- $\mathfrak{n}$: *modular derivation of A^δ*

Then

$$\boxed{\mathbf{n} = \mathbf{m} + \left(\sum_{i=1}^{n} \deg(x_i)\right) \delta - \mathrm{Div}(\delta)E.}$$

Unimodularity

Theorem (Tang-Zhang-W. 22)

- $(A = \Bbbk[x_1, \dots, x_n], \pi)$: *graded Poisson algebra*
- $\mathrm{Div}(E) = \deg(x_1) + \cdots + \deg(x_n) \neq 0$ *in* $\Bbbk$
- $\mathfrak{m}(-) = \mathrm{Div}(H_-)$: *modular derivation of A.*

Then $\left(A^{-\frac{1}{\mathrm{Div}(E)}\mathfrak{m}}, \pi_{unim}\right)$ *is unimodular and*

$$\boxed{\pi = \pi_{unim} + \frac{1}{\mathrm{Div}(E)}\left(E \wedge \mathfrak{m}\right).}$$

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- $rgt(A) = 0 \Rightarrow A$ *is unimodular*
- $rgt(A) = -1 \Rightarrow \dim_{\Bbbk} Gspd(A) = \dim_{\Bbbk} Gpd(A) = 2$

rgt for $\mathbb{k}[x, y, z]$

- $A = \mathbb{k}[x, y, z]$ with $\deg(x, y, z) = 1$

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Proposition (Tang-Zhang-W. 22)

Ω	0	x^3	x^2y	xyz	$xy(x+y)$	$xyz + x^3$	$xy^2 + z^2z$	$irred.$
$rgt(A)$	-8	-5	-3	-2	-2	-1	-1	0

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- $rgt(A(n,a)) = -1$

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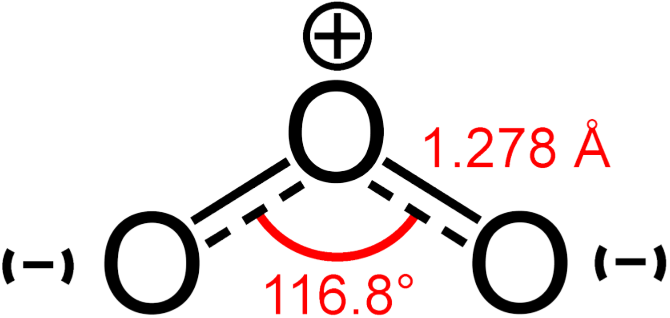

H-ozoness

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- A is H-ozone if every ozone Poisson derivation is Hamiltonian
- A is PH^1-minimal if $PH^1(A) \cong ZE$

H-ozoness

Theorem (Tang-Zhang-W. 22)

$$\begin{array}{ccc} A \text{ is } PH^1\text{-minimal} & \Longrightarrow & A \text{ is } H\text{-ozone} \\ \Downarrow & & \Downarrow \\ rgt(A) = 0 & \Longrightarrow & A \text{ is unimodular} \end{array}$$

Theorem (Tang-Zhang-W. 22)

$A = \Bbbk[x, y, z]$ *with Poisson center* Z.

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which are further equivalent to:

(4) *Any graded twist of A is isomorphic to A.*

(5) $h_{Pder(A)}(t) = \frac{1}{(1-t)^3}$.

(6) $h_{PH^1(A)}(t) = \frac{1}{1-t^3}$.

(7) $h_{PH^1(A)}(t) = h_Z(t)$.

(8) $h_{PH^3(A)}(t) - h_{PH^2(A)}(t) = t^{-3}$.

(9) *A is unimodular with irreducible Ω.*

Poisson cohomology of $\mathbb{k}[x, y, z]$

Corollary (Tang-Zhang-W. 22)

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$A = \Bbbk[x, y, z]$ unimodular quadratic Poisson algebra with irreducible potential Ω. Then

(1) $h_{PH^0(A)}(t) = \frac{1}{1-t^3}$

(2) $h_{PH^1(A)}(t) = \frac{1}{1-t^3}$

(3) $h_{PH^2(A)}(t) = \frac{1}{t^3}\left(\frac{(1+t)^3}{1-t^3} - 1\right)$

(4) $h_{PH^3(A)}(t) = \frac{(1+t)^3}{t^3(1-t^3)}$

Thank You!

Happy Birthday,

Paul!